Selected Readings in Micro: Session 4

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A Simple Example of Bilateral Contracting

- borrowed from Segal and Whinston ("Robust Predictions for Bilateral Contracting with Externalities," 2003, *Econometrica*)
- focus on vertical contracting (Hart and Tirole, 1990; McAfee and Schwartz, 1994)
  - one manufacturer sells to $N \geq 2$ retailers
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- retailers transform each unit of the manufacturer's product into one unit of final good at zero marginal cost and sell to consumers
- manufacturer's cost function

$$c(X) = \alpha X + \frac{1}{2}\beta X^2$$

- downstream inverse demand function

$$P(X) = \max\{a - bX, 0\}$$

Benchmark Cases

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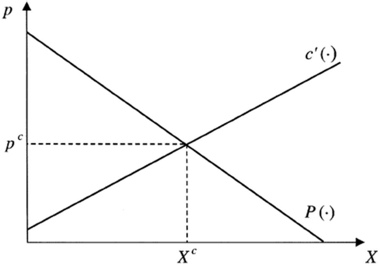
- "competitive" quantity (the vertical structure takes the retail price as given, i.e., retailers compete with each other)

$$P(X^c) = c'(X^c) \Rightarrow X^c = \frac{a-\alpha}{b+\beta} > X^*$$



Contracting Game

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- each retailer contracts with the manufacturer *without* observing other retailers' contracts with or purchases from the manufacturer
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- consider the "offer game": manufacturer makes simultaneous offers to the downstream firms, who then accept or reject
  - initially the manufacturer offers each retailer i a "point" contract (x_i, t_i), where $x_i \geq 0$ is the quantity of the product and t_i is the retailer's payment
  - after observing the offer, retailer i forms beliefs about other retailers' contracts

Passive Beliefs

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- let $\left(\hat{x}_1, ..., \hat{x}_N, \hat{t}_1, ..., \hat{t}_N\right)$ denote the equilibrium outcome, if retailer i is offered $(x_i, t_i) \neq \left(\hat{x}_i, \hat{t}_i\right)$, he still believes that other retailers make their equilibrium purchases $\hat{x}_{-i}$ and he will accept the offer if and only if $P\left(\hat{X}_{-i} + x_i\right) x_i \geq t_i$, where $\hat{X}_{-i} = \sum_{j \neq i} \hat{x}_i$

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- manufacturer's equilibrium sale $\hat{x}_i$ to i must be *pairwise stable*, i.e.

$$\hat{x}_i \in \arg\max_{x_i \geq 0} \left[P\left(\hat{X}_{-i} + x_i\right) x_i - c\left(\hat{X}_{-i} + x_i\right)\right]$$

  - interpretation: it is impossible to increase bilateral surplus between the manufacturer and any retailer i given the purchases of all other retailers

Pairwise-Stable Trades

- in this example, the unique profile of pairwise-stable trades is the symmetric profile

$$x^p = \left(\frac{\hat{X}_N^p}{N}, ..., \frac{\hat{X}_N^p}{N} \right),$$

where the aggregate quantity is

$$\hat{X}_N^p = \frac{a - \alpha}{(1 + 1/N)\, b + \beta}$$

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- basic patterns
  - $\hat{X}_1^p = X^*$(the efficient quantity), $\hat{X}_N^p > X^*$ for $N > 1$, i.e., inefficiency caused by externalities
  - $\hat{X}_N^p < X^c$ (the competitive quantity) for all N
  - $\hat{X}_N^p \to X^c$ as $N \to \infty$, i.e., competitive convergence

Problems with Passive Beliefs

- retailers should be aware that the manufacturer's optimal contract offer to one retailer depends on her contracts with other retailers (externalities)
  - an extreme example: suppose the manufacturer has only $\bar{X}$ units for sale, then a retailer who is offered $\bar{X}$ units can be sure that other retailers get none of the good, regardless of what the equilibrium allocation was supposed to be

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- once retailers are allowed to hold arbitrary beliefs after observing out-of-equilibrium offers, a large set of outcomes can be sustained with some equilibrium notion
  - for example, as noted by McAfee and Schwartz (1994), the efficient outcome X^*can be sustained for any N with *symmetry belief*, i.e., each retailer believes that the manufacturer offers the same contract to all retailers

Sensitivity Problem

- if we changed the information assumptions so that offers were public (or there were some other way to commit to what it sends to both firms), or if payment would only be made after the second period and would be contingent on returns in the second period, the results are quite different

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- if we changed the information assumptions so that offers were public (or there were some other way to commit to what it sends to both firms), or if payment would only be made after the second period and would be contingent on returns in the second period, the results are quite different
- main problem: results seem sensitive to seemingly minor change in timing or informational assumptions

Buyer Power and Bargaining

- we now move to models where the upstream firms don't just dictate the terms of trade to downstream firms
  - Coca Cola doesn't go to Walmart and say here is our price, take it or leave it
  - on the other hand Walmart itself might tell a small manufacturer: "we will carry your product for price x, take it or leave it"

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- which model is relevant depends on the institutions we are trying to model
- equilibrium in the model with bargaining will satisfy Nash like constraints with an assumption about off the equilibrium path beliefs, but mostly sweeps modeling of informational issues under the rug

Bargaining Problem

- a bargaining problem is when two agents can create a surplus together, but they disagree over how to split the surplus
- simplest case is I have a product to sell to you, the surplus is the difference in our values, I would like to charge your full valuation, you would like to pay just may reservation value
- any price weakly between these two is better than no transaction
- basic question: what price gets chosen?

Nash Bargaining

- set of feasible allocations: X
  - simplest case: $X = \{(x_1, x_2) : x_1 + x_2 = 1\}$
- two players with utility functions on X: $u_i(x)$
- disagreement values: if they disagree and do not contract each player gets d_i
- bargaining parameters b: a measure of bargaining power

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- Nash bargaining solution

$$\arg\max_{x \in X} \left[u(x) - u(d_1)\right]^b \left[u(1-x) - u(d_2)\right]^{1-b}$$

  - somewhat incomplete because there is no determination of what the disagreement payoff is (value of the outside option)

Theory Rational

- Rubinstein shows that the limit of an alternating move game (one person makes offer, other either accepts or makes counteroffer, ...) with time discounting and transferable utility as the time interval between offer and counteroffer is taken to zero, has a unique limit which is the Nash bargaining solution

Horn and Wolinsky: Nash-in-Nash

- Horn and Wolinsky ("Bilateral Monopolies and Incentives for Merger," 1988, RAND) model a vertical market as a group of interrelated Nash bargains
  - equilibrium notion is going to be a Nash equilibrium between Nash bargaining situations

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  - equilibrium notion is going to be a Nash equilibrium between Nash bargaining situations
- the Nash equilibrium ensures that no players perceives that when a player is in a contractual relationship it cannot do better by breaking the relationship than by being in the relationship (and visa versa if it were not in a relationship)
  - to ensure that this is the case, we need a model of perceptions of what would happen if an extant relationship were broken or a relationship that did not exist were formed

Setup

- players
  - N downstream firms, $n = 1, ..., N$
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- technology
  - upstream firms produce inputs
  - downstream firms procure inputs and product final goods for sale to consumers

Payoffs and Strategies

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  - can of course assume Bertrand competition in non-linear prices, upstream or downstream

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  - can of course assume Bertrand competition in non-linear prices, upstream or downstream
- the alternative being used in empirical work is Nash bargaining in linear prices for each (buyer-seller) couple and Bertrand competition downstream

Nash Bargaining in Linear Prices

- each upstream firm m and each downstream firm n negotiate a linear input fee $\tau_{m,n}$

$$\max_{\tau_{m,n}} \left[\Pi_n\left(\tau_{m,n}, \tau_{-(m,n)}\right) - \Pi_n\left(\emptyset, \tau_{-(m,n)}\right)\right]^b \left[\Pi_m\left(\tau_{m,n}, \tau_{-(m,n)}\right) - \Pi_m\left(\emptyset, \tau_{-(m,n)}\right)\right]^{1-b}$$

  - if n sells a single output and uses M inputs

$$\Pi_n\left(\tau_{m,n}, \tau_{-(m,n)}\right) = \left(p_n^* - \sum_{m=1}^{M} \tau_{m,n}\right) D_n\left(\tau_1, ..., \tau_n\right)$$

  - if each upstream firm sells to all downstream firms

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- this is bargaining with "externalities", because the profits of the upstream and downstream firms may well depend on the bargains made by third parties or by themselves with other parties

Some Notes

- disagreement values: Horn and Wolinsky (Nash-in-Nash) hold all other contracts fixed, there is a question of whether this makes sense in any given application, but if they do not, you have to replace them with something else

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- the payoffs are the Nash products and all moves are simultaneous (simultaneous move Nash equilibrium)
- calculation: solve numerically like any other static game, i.e., a fixed point for a system of FOCs

Interpretation of the Bargaining Parameters

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- in the Rubinstein models they represent relative differences in time discounting
- in practice, they are going to pick up factors which are un-modeled, e.g., possibility of backwards or forwards integration, possibility of another entrant entering one side or the other, negotiating skill, repeated bargaining and dynamic commitments or reputations, asymmetric information on primitives

Next Class Presentation

- Allan Collard-Wexler Robin Lee and Gautam Gowrisankaran: " "Nash-in-Nash" Bargaining: A Microfoundation for Applied Work," working paper, 2017.