Graduate IO: Problem Set #1

Due Date: Oct. 23 (in class)

October 8, 2017

1. Consider the classic Cournot model of quantity competition, with asymmetric marginal costs. Two firms $i \in \{1, 2\}$ simultaneously choose production levels $q_i \in R^+$; each firm i produces those units at a constant marginal cost of c_i per unit, and sells them at the market price, which is determined by the inverse demand function

$$P = \max\{0, 100 - q_1 - q_2\}.$$

   (a) Calculate firm i's best-response given a production level q_j of his opponent. (Be sure to account for the cases where it is optimal to set $q_i = 0$.)

   (b) Let $c_1 = 25$ and $c_2 = 55$. Find the Nash equilibrium in which both players produce, and calculate both firms' profits.

   (c) If firm 1 produced 45 units, firm 2's best-response would be not to produce at all. Calculate firm 1's profits in this event. Is it higher or lower than your answer to part 2? Is $(q_1, q_2) = (45, 0)$ an equilibrium? Why or why not?

2. Suppose there are two coffee houses along Main Street. The street is one mile long. One hundred residents are uniformly distributed along this stretch, and each resident purchases one cup of coffee per day. Cups of coffee differ only in their location and price, not in any other way. Each customer derives a utility of $v = \$3.00$ from a cup of coffee. Starbucks is located at either end of the one mile stretch, and Esquire Coffee is located halfway between the two endpoints of the street. The prices of coffee at Starbucks' two locations are p_0 and p_1, respectively, and Esquire's price of coffee is denoted by r. Marginal costs of a cup of coffee are zero. A consumer's cost of travel is quadratic in the distance from home to any of the coffee houses.

   (a) Determine the location of the two marginal consumers: the one who is indifferent between purchasing from Esquire and the Starbucks located at the left end point, and the one who is indifferent between purchasing from Esquire and the Starbucks located at the right end point, assuming that prices are such that they exist.

   (b) Derive the best reply functions to the pricing game in which the coffee houses choose prices simultaneously. Assume that Starbucks can set different prices at its two locations.

   (c) Determine the equilibrium prices and market shares.

   (d) Suppose Starbucks sells one of its coffee houses to Seattle Best Coffee. Derive the new equilibrium prices and market shares.

3. Consider the linear city model we talked about in class. Suppose the vendors play a two-stage game in which they first choose locations and then compete in prices.

(a) Assume the transport costs are quadratic in distance traveled (the case we discussed in class). Derive the subgame perfect equilibrium outcome, i.e., equilibrium location and prices.

(b) Suppose the transport cost is *linear* in distance traveled, i.e., transport cost is tx for consumer x, using the notation in the lecture slides. Derive the conditions under which the equilibrium exists and characterize the equilibrium outcome.

4. Consider a homogenous good market with two firms, 1 and 2. The two firms simultaneously choose capacities k_1 and k_2, and then, after observing each other's choice of capacity, compete for customers by setting prices. Cost per unit of capacity is c and production costs are zero. The market demand is given by $D(P) = 1$ if $P \leq 1$, and zero otherwise.

   (a) Characterize the sets of (k_1, k_2) for which there is a pure strategy Nash equilibrium in prices, and determine the equilibrium.

   (b) Derive closed form solutions for the mixed strategy equilibrium for the remaining set of capacity choices.

   (c) Derive the profit functions for the first stage under the assumption of Nash equilibrium play in the pricing stage, and show that they have the Cournot form.

   (d) Determine the set of subgame perfect capacity choices.