Graduate IO: Oligopolistic Competition in Homogenous Products Markets

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Oligopoly Pricing in Homogenous Products Markets

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- main question: how are prices and output determined when there are a small number of firms producing a homogenous (identical) product?
- for starters, we fix an aggregate demand curve and investigate how firms make decisions
- monopoly models: agent optimizes against a fixed environment
- oligopoly models: agent has to consider what actions its competitors will take and how they may react to its actions

Cournot Model

- normal form representation
  - player: $i = 1, 2, ..., N$
  - strategy for firm i: $q_i \in [0, \infty)$
  - payoff: $\pi(q_i, q_{-i}) = P\left(\sum_i q_i\right) q_i - C(q_i)$

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- examples: commodity markets (e.g., spring water, sugar)

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- interpretation: each firm is behaving optimally given its conjecture about its rival's choice of quantity and, in equilibrium, their conjectures are correct

Best-Reply Response Representation

- let $R_i(q_{-i})$ denote firm i's best replies to its rivals' output choices, then a Nash equilibrium is a profile $\{q_1^*, ..., q_N^*\}$ such that

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- in other words, the Nash equilibrium is a fixed point of the mapping $\mathbf{R}$, where $\mathbf{R}$ is the Cartesian product of R_i
- existence reduces to checking that $\mathbf{R}$ meets conditions of some fixed point theorem, this also provides algorithm for finding Nash equilibria

Symmetric Case with Linear Demand

- demand curve: $P(Q) = a - bQ, \; Q = q_1 + ... + q_N$
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- taking first-order condition, the best reply is

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- solve the N equations for N unknown

$$q_i^* = \frac{a - c}{b(N+1)}$$

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$$R\left(q_j\right) \equiv q_i = \frac{a - c_i - bq_j}{2b}$$

- solution (equilibrium)

$$q_i^* = \frac{a - 2c_i + c_j}{3b}$$

  - implication: the more efficient (lower cost) firm produces more output, or large firms have lower marginal cost in equilibrium

Markup and Concentration

- evaluated at the equilibrium, the FOC can be written as the Lerner's index (percentage markup)

$$\frac{P\left(Q^{*}\right)-c_i}{P\left(Q^{*}\right)}=\frac{s_i^{*}}{\eta}$$

where $s_i^{*}=\frac{q_i^{*}}{Q^{*}}$ is firm i's market share and η is the the elasticity of market demand

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- consequently, average price cost margin is

$$\sum_i s_i \frac{P(Q^*) - c_i}{P(Q^*)} = \sum_i \frac{s_i^2}{\eta} = \frac{HHI}{\eta}$$

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- some implications
  - firms have market power (price is above marginal cost): solution lies between competition and monopoly
    - more firms $\Rightarrow$ smaller shares $\Rightarrow$ lower markups
  - the more elastic is demand, the lower are the markups
  - firms with lower marginal costs have higher market shares

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  - firms' products are perfect substitutes: consumers buy from firm offering the lowest price
- players: two firms indexed by $i = 1, 2$
- strategy of firm i: $p_i \in [0, \infty)$
- payoffs function

$$\pi_i(p_i, p_j) = \begin{cases} p_i Q(p_i) - C(Q(p_i)) & \text{if } p_i < p_j \\ \frac{1}{2}\left[p_i Q(p_i) - C(Q(p_i))\right] & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

Solution

- Nash Equilibrium: a pair of prices $\{p_1^*, p_2^*\}$ such that

$$\pi_i \left(p_i^*, p_j^*\right) \geq \pi_i \left(p_i, p_j^*\right) \ \forall p_i \in [0, \infty) \ \ i = 1, 2$$

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- unique NE: $p_1^* = p_2^* = c$
  - can $p_1 > p_2 > c$ be an equilibrium?
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  - no, because it is profitable to undercut rival

Implication: Bertrand Paradox

- puzzle: one is monopoly, two is perfect competition?
  - in reality, firms do not typically sell at marginal costs in markets with few sellers
  - with asymmetric mc, the industry should be a monopoly and thus $p \neq mc$, thus we should never see such equilibrium

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- price competition seems more natural than quantity competition but yields predictions that contradict reality, here are some possible issues with the underlying assumptions
  - unlimited capacity: in reality firms may not have the capacity to serve the whole market
  - homogeneous good: in reality it's rare for two firms' products to be perfect substitutes
  - static competition: in reality firms play pricing games against each other repeatedly
  - perfect information: in reality consumers may have to engage in costly search to determine which firm has the lowest price

Reading for Next Class

- D. Kreps and J. Scheinkman, "Quantity Precommittment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 1983.