Graduate IO: Session 7

October 31, 2016

About the Presentations

- each group should prepare for a one-hour presentation
- slides must be in English; use English whenever possible when you present
- be clear and concise
- practice (at least) 3 times in advance

Agenda

- a simple model of entry: Mankiw and Whinston (1986)
- empirical models of entry: Bresnahan-Reiss approach and its extensions

Free Entry: Mankiw and Whinston (1986)

- goal: characterize the conditions under which the number of entrants in a free-entry equilibrium is excessive, insufficient or optimal

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- goal: characterize the conditions under which the number of entrants in a free-entry equilibrium is excessive, insufficient or optimal
- optimality: social planner chooses welfare maximizing number of firms subject to the constraint that it cannot control behavior of firms in the post-entry game, i.e., focus on entry behavior

Homogenous Good Market

- inverse demand: $P(Q), P' < 0$
- entry costs: K
- identical firms with cost function $c(q)$ that is continuous, non-decreasing and convex
- q_N is the *symmetric* equilibrium output per firm in the post-entry game with N firms
- $\pi_N = P(Nq_N)\, q_N - c(q_N) - K$

Solutions

- socially optimal number of firms N^* solves the following problem

$$\max_N W(N) = \int_0^{Nq_N} P(s)\,ds - Nc(q_N) - NK$$

  - FOC: $W^{'}(N^*) = 0$

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  - FOC: $W'(N^*) = 0$

- free entry: N^e solves $\pi_N = 0$

Main Assumptions

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- $q_N < q_{N'}$ for all $N > N'$
  - output per firm falls with number of firms
- $P(q_N) - c'(q_N) \geq 0$ for all N
  - equilibrium price cannot fall below marginal cost

Excessive Entry

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- proof
  - differentiating $W(N)$ and rearranging terms

$$W^{'}(N) = \pi_N + N\left[P(Nq_N) - c^{'}(q_N)\right]\frac{\partial q_N}{\partial N},$$

    given our assumptions, the second term on the RHS is non-positive, therefore, $W^{'}(N) \leq \pi_N$ for all N, which implies $\pi_{N^*} \geq W^{'}(N^*) = 0$
  - to complete the argument, we need to show that profits per firm fall as N increases

$$\frac{\partial \pi_N}{\partial N} = \left[P(Nq_N) - c^{'}(q_N)\right]\frac{\partial q_N}{\partial N} + q_N P^{'}(Nq_N)\frac{\partial (Nq_N)}{\partial N} < 0$$

    the inequality follows from our assumptions, hence, $N^e \geq N^*$ because $\pi_{N^e} = 0$ (zero-profit condition)

Interpretation

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- the response of existing firms is to contract their output, this contraction is known as the "business-stealing" effect
- in choosing whether or not to enter, the entrant does not take into account the negative impact of its entry on profits of existing firms: excessive entry

Differentiated Markets

- consumer benefits: $G\left[\sum_i f(q_i)\right]$
  - q_i is firm i's output
  - $f(0) = 0$, $f^{'} > 0$ and $f^{''} \leq 0$ for all $q \geq 0$
  - $G^{'}(z) > 0$, $G^{''}(z) < 0$

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- these assumptions imply that consumers prefer variety, and outputs of firms are substitutes

Solving the Model

- social planner problem

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- on the other hand, in post-entry symmetric equilibrium
  - each firm's price is equal to $G' f'$
  - thus, firm's profit: $\pi_N = G' f' q_N - c(q_N) - K$

Solving the Model (Cont.)

- re-arranging,
  $W^{'}(N) = \pi_N + N\left[G^{'}f^{'} - c^{'}\right]\frac{\partial q_N}{\partial N} + G^{'}\cdot\left[f - f^{'}q_N\right]$
  - the first term measures the direct effect of the entrant on social surplus
  - the second term measures the business-stealing effect, which reduces social surplus
  - the third term captures the positive effect of increased product diversity on social surplus

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- the diversity and business-stealing effects go in opposite direction so direction of entry bias is ambiguous
  - depends on whether $\pi_{N^*} \geq W^{'}(N^*) = 0$ or $\pi_{N^*} \leq W^{'}(N^*) = 0$

Empirical Models of Entry

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- a firm enters a market if it expects to earn positive profits net of entry costs
- empirical objective: estimate entry costs and the properties of the reduced form, post-entry profit function
  - in particular, how does the profit function depend upon N, the number of firms in the market?
  - post-entry profits should decline with N but rate of decline differs depending upon the nature of post-entry competition

Model I: Symmetric Firms

- data: cross-section of markets $\{N_t, S_t, X_t\}_{t=1}^{T}$
  - N_t: number of firms in market t
  - S_t: size of market t
  - X_t: vector of profit shifters

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- firms are identical so the value of entering market with N_t firms is the same for all firms

$$\Pi(N_t) = W(N_t, S_t, X_t; \theta) - F_t$$

  - W is the continuation value function from entry - present discounted value of post-entry profits
  - F_t is the fixed cost of entering market t

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- suppose there are two potential entrants, $j = 1, 2$ and $\Pi(2) < 0 < \Pi(1)$
- the entry game has two pure strategy Nash equilibria: either firm 1 enters or firm 2 enters
- thus, the mapping from the unobserved fixed costs of entry into outcomes is not one-to-one, this fact implies that likelihood function is not well defined

Multiple Equilibria

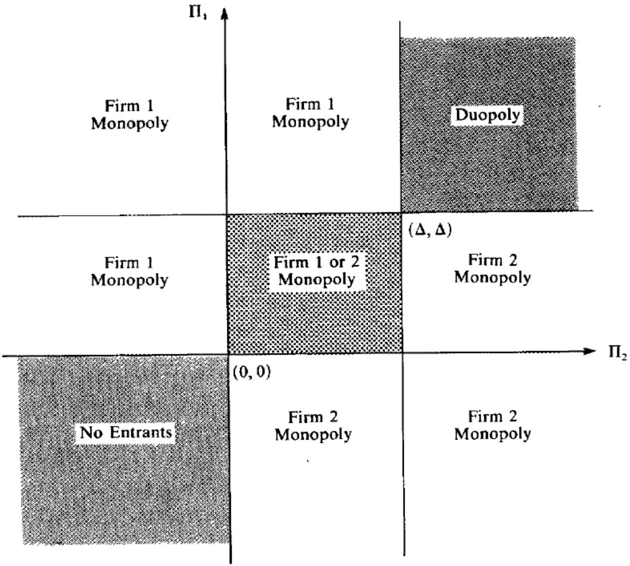

FIGURE 3

Bresnahan and Reiss Approach

- re-define the endogenous variable as the number of firms rather than entry decision of individual firms

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- also, restrict analysis to pure strategy Nash equilibria, P.N.E. implies that N_t satisfies

$$\Pi\left(N_t, S_t, X_t; \theta\right) > 0 > \Pi\left(N_t + 1, S_t, X_t; \theta\right)$$

or equivalently,

$$W\left(N_t, S_t, X_t; \theta\right) > F_t > W\left(N_t + 1, S_t, X_t; \theta\right)$$

Empirical Strategy

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- assume that F is distributed normal with mean $\bar{F}$ and variance σ, then the probability of observing n firms in market t is given by

$$\begin{aligned} P_{nt} &= \Pr\left(N_t = n\right) \\ &= \Phi\left[W\left(n, S_t, X_t; \theta\right) - \bar{F}, \sigma\right] \\ &- \Phi\left[W\left(n+1, S_t, X_t; \theta\right) - \bar{F}, \sigma\right] \end{aligned}$$

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- the parameter estimates determine a vector of cutoff points in market t

$$W_t\left(1\right) > W_t\left(2\right) > ... > W_t\left(N\right)$$

  where $W_t\left(n\right)$ is the value of fixed cost that makes the n^{th} firm just indifferent between entering or staying out

Market Size

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- fix the values of F and X at their sample means, $\bar{F}$ and $\bar{X}$

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- fix the values of F and X at their sample means, $\bar{F}$ and $\bar{X}$
- define S^N as the population size which makes the N^{th} firm indifferent between entering or staying out, it solves

$$W\left(N, S^N, \bar{X}; \theta\right) = \bar{F}$$

- the vector of cutoffs $\left\{S^1, ..., S^N\right\}$ are called entry thresholds, clearly, they are increasing since W is decreasing in N and increasing in S

Per Firm Thresholds

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- answer depends upon post-entry behavior and, to some extent, functional form assumptions, BR assume

$$W(N, S, X; \theta) = V(N, X; \theta) S$$

- thus,

$$S^N = \frac{\bar{F}}{V(N, X^*; \theta)}$$

Per Firm Thresholds (Cont.)

- suppose firms collude on monopoly price and good is homogeneous, then

$$V(N) = \frac{V(1)}{N} \, \forall N$$

  - note that $s^N = \frac{\bar{F}}{NV(N)}$ and $s^{N+1} = \frac{\bar{F}}{(N+1)V(N+1)}$
  - hence, $s^N = s^{N+1}$

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- therefore, if the per firm thresholds are decreasing in N, that is, $s^{N+1} > s^N$, then this is evidence of price competition

Product Differentiation

- product differentiation complicates the story since in this case entry also expands the market,
  - in the polar case of no substitutability, $V(N)$ is independent of N, which implies entry thresholds are a constant, i.e., $s^N = s^{N+1}$

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- basic point: cannot distinguish between collusion and product differentiation

BR Sample Design

- cross-section of geographical markets with different populations
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- they have to be small enough to support no more than a few suppliers, in RES paper the number was 2; in later JPE paper, it is larger - here is a table from the JPE paper

	Entry Thresholds (000's)					Per Firm Entry Threshold Ratios			
Profession	S_1	S_2	S_3	S_4	S_5	s_2/s_1	s_3/s_2	s_4/s_3	s_5/s_4
Doctors	.88	3.49	5.78	7.72	9.14	1.98	1.10	1.00	.95
Dentists	.71	2.54	4.18	5.43	6.41	1.78	.79	.97	.94
Druggists	.53	2.12	5.04	7.67	9.39	1.99	1.58	1.14	.98
Plumbers	1.43	3.02	4.53	6.20	7.47	1.06	1.00	1.02	.96
Tire dealers	.49	1.78	3.41	4.74	6.10	1.81	1.28	1.04	1.03

Remarks

- for doctors, dentists, tire dealers, thresholds decline a lot in moving from 1 to 2 but not much thereafter - two or three firms are enough to achieve competitive outcomes
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- plumber's prices never fall
- homogeneity assumption is quite strong: a threshold near 1 could simply reflect the offsetting effects of market expansion and competition
- would like to observe prices, BR do observe prices for tire dealers and find that they fall with the first few entrants and then level off, which is consistent with the threshold size pattern

Extensions

- entry and exit thresholds are likely to depend upon population growth, BR consider this factor by letting market size depend upon factors other than town population

$$S(Y_t) = TownPop_t + \lambda_1 NGRW_t + \lambda_2 PGRW_t + ...$$

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- mean of the distribution of fixed cost varies with N, the idea is that distribution of entry costs for the first entrant is different than for the second entrant

Extensions (Cont.)

- Genesove (2002) uses the BR model to study competition in daily newspaper industry, he has panel data on the population of towns and on their number of daily papers, he uses a different econometric model to estimate the thresholds

$$Q_{nt} = \Pr\left\{ N_t \geq n \middle| S_t; \theta, \sigma \right\} = \Phi\left[W\left(n, S_t; \theta\right); \sigma \right]$$

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- Berry and Waldfogel consider entry into the radio industry where both price and quantity data are available
  - additional information allows the authors to estimate some of the parameters in $W\left(N, S, X\right)$
  - entry data are used to estimate any remaining parameters and distribution of F

Extensions (Cont.)

- Mazzeo extends the BR model in a different direction
  - firms can choose to enter with a high or low quality product, the application is motels
  - thus, W depends upon N, number of firms offering low quality products and M, number of firms offering high quality products
  - zero profit condition is more complicated because need to consider two continuation value functions: W_{1t} and W_{2t} that measure returns to entry with low and high quality products

$$W_{it} = \beta_i X_t + g\left(M_t, N_t, \theta\right) + \xi_{it}, i = H, L$$

where the unobserved error is same for all firms but different for high quality firms than low quality firms

Extensions (Cont.)

- Mazzeo's extension (cont.)
  - firms are symmetric ex-ante and symmetric ex-post conditional on quality choice, they make their choices sequentially so equilibrium is unique
  - need to partition the space (ξ_1, ξ_2) into regions where equilibrium is (n, m), compute probabilities by simulation

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- Berry considers the case of heterogeneous firms

$$\Pi(N_t) = W(N_t, X_t; \theta) - F(Z_{ft}, \xi_{ft}; \theta)$$

where Z_{ft} are firm-market specific characteristics and ξ_{ft} is a firm-market unobservable affected entry costs

Berry (1992) (Cont.)

- because the firm-specific characteristics affect entry costs but not the continuation value, there is a unique (pure strategy) equilibrium number of firms, this allows Berry to use the B-R approach for computing probability of the event that $N_t = n$ and finding parameter values that minimize the likelihood function
  - main problem, however, is that the event probabilities involve high dimensional integrals - as many integrals as there are firms
- use simulation methods to compute the relevant probabilities and method of moments (match predicted N with the observed N) to estimate the parameters
- Berry's application is airline markets

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  - allows for mixed strategy equilibria, flexible correlation of unobservables and heterogeneous rivals
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- the authors use the framework to study entry in airport-pair markets

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- the equilibrium mapping from the primitives of the model to outcomes is a correspondence, not a function
  - given the primitives, the model does not generate a unique reduced form (i.e., distribution for outcomes)
- previous studies modify the model so that it yields a well-defined likelihood function for the data
  - restrict game to two players and exploit the fact that probability of events {no entry} and {both entry} are well-defined
  - restrict the payoffs and focus on uniqueness of number of entrants
  - change the game form to obtain uniqueness: sequential entry (selects one of the asymmetric pure strategy equilibria) or private information (selects the mixed strategy equilibrium)

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  - estimate the empirical distribution of outcomes conditional on market characteristics
  - choose parameter values to minimize the distance between this distribution and the bounds
- cost: point-identification is typically not obtained, the strategy identifies a class of models that are consistent with the data

The Model

- markets: $m = 1, ..., M$
- players: $i = 1, ..., K$
- strategy space: $y_i \in \{0, 1\}^M$
- payoffs

$$\Pi_i \left(y_i, y_{-i}\right) = \sum_{m=1}^{M} \pi_{im} \left(y_{im}, y_{-im}\right)$$

$$\pi_{im} \left(y_{im}, y_{-im}\right) = \alpha_{i0} + \alpha_{ix} \log \left(X_m\right) + \sum_{j \neq i} \delta_{ij} y_{jm} + \epsilon_{im}$$

The Model (Cont.)

- best reply

$$y_{im} = 1\left(\alpha_{i0} + \alpha_{ix}\log\left(X_m\right) + \sum_{j\neq i}\delta_{ij}y_{jm} + \epsilon_{im} \geq 0\right)$$

The Model (Cont.)

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- assumption: $(\epsilon_{1m}, ..., \epsilon_{Km})$ is random draw from distribution F with mean zero, covariance Ω

- let $l = 1, ..., 2^K$ index the set of possible outcomes (drop subscript m)

$$y_1 = (0, ..., 0)$$

$$y_2 = (0, 1, ..., 0)$$

$$y_{2^K} = (1, ..., 1)$$

Bounds

- let $\theta = (\alpha, \delta, \Omega)$, define the bounds on the outcome probabilities (conditional on x, θ) as

$$H_i(x, \theta) = \begin{bmatrix} H_i(y_1; x, \theta) \\ \vdots \\ H_i(y_{2^K}; x, \theta) \end{bmatrix}, \ i \in \{H, L\}$$

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- if model is correct, observed frequencies lie between these bounds (at true θ_0)
- simulation: fix θ, x and take R random draws from F

Bounds (Cont.)

- for each draw, calculate the profits of each player for all $l = 1, ..., 2^K$ outcomes and determine the set of equilibrium outcomes

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- then
  - $\hat{H}_U(y_l; x, \theta)$ is fraction of times y_l is an equilibrium
  - $\hat{H}_L(y_l; x, \theta)$ is fraction of times y_l is the only equilibrium

Estimation

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- F and unknown selection rules determines an empirical distribution of outcomes
  - obtain a consistent nonparametric estimator of the distribution
- construct an estimator that minimizes the violations of the inequality constraints: empirical distribution outcomes should lie between the predicted bounds

Application

- entry into airport-pair markets in May 1998
  - data drawn from T-100 domestic segment, which contains domestic non-stop segment data by carrier
  - a carrier enters an airport-pair market if it offers non-stop flights in that market

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  - data drawn from T-100 domestic segment, which contains domestic non-stop segment data by carrier
  - a carrier enters an airport-pair market if it offers non-stop flights in that market
- markets: 40 airports, 766 airport-pairs
  - characteristics: log of distance, log of product of populations served by the airports
  - players: American (AA), Delta (DL), Southwest, etc.

Main Findings

- errors are correlated across carriers within a market: correlation matters a lot
- predictive power is not great, only 35%
- rivals are quite heterogeneous
- for example, Delta's presence increases the probability that American enters, and vice versa

Reading for Next Class

- Alan Sorensen, "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs", *Journal of Political Economy*, 2000.