Graduate IO: Session 5

October 16, 2016

Agenda

- estimation algorithm of BLP: MSM
  - Nevo (2000), Knittel and Metaxoglou (2014)
- new estimation strategy: Lu, Shi and Tao (2016)
  - application to housing market
- zero problem: Gandhi Lu and Shi (2016)

Setup

- $x_j = (x_{j,1}, ..., x_{j,K})$ is a vector of K observable characteristics of product j
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- utility function

$$u_{ij} = x_j \beta_i - \alpha p_j + \xi_j + \varepsilon_{ij}$$

Random Coefficients

- the K random coefficients are

$$\beta_{i,k} = \beta_k + \sigma_k \eta_{i,k}$$

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- now we can write the utility function as

$$u_{ij} = \delta_j + \upsilon_{ij}$$

Market Share and Moment Condition

- the predicted market share of product j

$$s_j(\delta, \sigma) = \int \frac{\exp\left(\delta_j + \sum_k x_{j,k}\eta_{i,k}\sigma_k\right)}{1 + \sum_m \exp\left(\delta_m + \sum_k x_{m,k}\eta_{i,k}\sigma_k\right)} d\Phi\left(\eta_i\right)$$

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- moment condition for GMM

$$E[\xi_j | z_j] = 0 \ \forall j \ a.s.$$

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- find the value of ξ and minimize the GMM objective function

$$\left[\frac{1}{J}\sum_{j=1}^{J} z_j^{'}\left(\delta_j\left(\sigma\right) - x_j\beta + \alpha p_j\right)\right] W_J \left[\frac{1}{J}\sum_{j=1}^{J} z_j\left(\delta_j\left(\sigma\right) - x_j\beta + \alpha p_j\right)\right]$$

New Estimation Strategy

- Lu Shi and Tao (2016): "A Semi-nonparametric Estimator for Random Coefficient Logit Demand Models"

Zero Problem

- Gandhi Lu and Shi (2016)

Reading for Next Class

- Brian McManus, “Nonlinear pricing in an oligopoly market: the case of specialty coffee”, *The RAND Journal of Economics*, 2007