

## Graduate IO: Session 2

September 19, 2016

## Before Start

- ▶ important: join the WeChat group (required if you are enrolled)

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- ▶ important: join the WeChat group (required if you are enrolled)
- ▶ group for the presentation/referee report
  - ▶ in principle 2 students
  - ▶ send me your group before next class

# Agenda

- ▶ product differentiation: basic models
  - ▶ two-product model
  - ▶ turnpike model
- ▶ horizontal differentiation
  - ▶ linear city model
  - ▶ circular city model
  - ▶ Dixit-Stiglitz-Spence Model

# Product Differentiation

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- ▶ most markets are differentiated product markets: products of different firms are not perfect substitutes
- ▶ our models of these markets are among the most realistic and useful of all models in IO
- ▶ two class of models: horizontal differentiation and vertical differentiation

## Horizontal Differentiation

- ▶ if all products were priced the same, consumers would disagree on which is the most preferred product



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- ▶ key attributes on which consumers differ: product characteristics or location

# Horizontal Differentiation

- ▶ if all products were priced the same, consumers would disagree on which is the most preferred product
- ▶ key attributes on which consumers differ: product characteristics or location
- ▶ examples: films, beer, cars, books, cereals, ice cream flavors, apartments/houses, Starbucks by geographic location

## Vertical Differentiation

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# Vertical Differentiation

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- ▶ key attributes: quality, durability
- ▶ examples: computers, diamonds, batteries
- ▶ in general, a product is a bundle of vertical and horizontal attributes: a car is a bundle of certain amount of horsepower (vertical), color, weight, and size (horizontal)

# A Basic Model of Product Differentiation

- ▶ a market with two products with demand functions

$$q_1(p_1, p_2) = a - bp_1 + cp_2$$

$$q_2(p_1, p_2) = a - bp_2 + cp_1$$

- ▶ two goods are substitutes if  $c > 0$ , complement if  $c < 0$
- ▶ own-price effect has to be larger than cross-price effect:  $b > |c|$

# A Basic Model of Product Differentiation

- ▶ a market with two products with demand functions

$$q_1(p_1, p_2) = a - bp_1 + cp_2$$

$$q_2(p_1, p_2) = a - bp_2 + cp_1$$

- ▶ two goods are substitutes if  $c > 0$ , complement if  $c < 0$
- ▶ own-price effect has to be larger than cross-price effect:  $b > |c|$

- ▶ invert the demand system

$$p_1(q_1, q_2) = \alpha - \beta q_1 - \gamma q_2$$

$$p_2(q_1, q_2) = \alpha - \beta q_2 - \gamma q_1$$

where  $a = \frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}$ ,  $b = \frac{\beta}{\beta^2-\gamma^2}$  and  $c = \frac{\gamma}{\beta^2-\gamma^2}$



## Pricing Game

- ▶ firm 1 chooses price to maximize

$$\pi_1(p_1, p_2) = (a - bp_1 + cp_2)p_1$$

and we can obtain

$$p_1 = \frac{a + cp_2}{2b}$$

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- ▶ similarly, for firm 2

$$p_2 = \frac{a + cp_1}{2b}$$

- ▶ solving for the equilibrium

$$p_1^* = p_2^* = \frac{a}{2b - c}$$

# Quantity Game

- ▶ firm 1 chooses quantity to maximize

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- ▶ best replies

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- ▶ best replies

$$q_1 = \frac{\alpha - \gamma q_2}{2\beta}, \quad q_2 = \frac{\alpha - \gamma q_1}{2\beta}$$

- ▶ equilibrium

$$q_1^* = q_2^* = \frac{\alpha}{2\beta + \gamma}$$

# Pricing Game VS Quantity Game

- ▶ basic question: how do quantities and prices differ in the pricing game versus the quantity game?

# Pricing Game VS Quantity Game

- ▶ basic question: how do quantities and prices differ in the pricing game versus the quantity game?
- ▶ answer: substituting for  $a$ ,  $b$  and  $c$ , one can show that price is higher (and quantity is lower) in the quantity game than in the pricing game
  - ▶ the difference is smaller when products are more differentiated, i.e.,  $\gamma$  goes to zero
  - ▶ what happens if  $\gamma = \beta$  (or  $b = c$ )?



# Strategic Complements VS Substitutes

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- ▶ best replies are strategic complements if they slope up, strategic substitutes if they slope down
- ▶ slope of the best reply functions is a key issue in many competitive situations

# The Turnpike Model

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# The Turnpike Model

- ▶ this model illustrates why antitrust policy in markets with complements is fundamentally different than in markets with substitutes
- ▶ model setup
  - ▶ one mile private road from A to B
  - ▶  $N$  individuals own segments of the road, each owner installs a toll booth to collect a toll for her segment
  - ▶ traffic demand of the road:  $D(P) = a - bP$ , where  $P = \sum_{i=1}^N p_i$
  - ▶ costs are zero and owners set tolls simultaneously

# Equilibrium

- ▶ owner  $i$  chooses  $p_i$  to maximize

$$\pi(p_i, p_{-i}) = p_i \left( a - bp_i - b \sum_{i \neq j} p_j \right)$$

- ▶ differentiating and solving for  $p_i$

$$p_i = \frac{a - b \sum_{i \neq j} p_j}{2b}$$

- ▶ symmetric equilibrium

$$p_i^* = \frac{a}{(N+1)b} \quad \forall i$$

$$P^* = \frac{aN}{(N+1)b}, \quad Q^* = \frac{a}{N+1}, \quad \pi^* = \frac{a^2}{b(N+1)^2}$$

# Implications

- ▶ what happens as  $N$  gets large? i.e., more and more entrants
  - ▶  $P^*$  increases to  $\frac{a}{b}$ ,  $Q^*$  goes to zero: price is lowest when there is only one firm

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- ▶ what happens as  $N$  gets large? i.e., more and more entrants
  - ▶  $P^*$  increases to  $\frac{a}{b}$ ,  $Q^*$  goes to zero: price is lowest when there is only one firm
- ▶ conclusion: consumers are better off with a monopoly than competition!
  - ▶ think of  $Q(P)$  as the size of the pie and  $p_i Q(P)$  as individual  $i$ 's share of the pie
  - ▶ at the monopoly price, the first-order effect of an increase in  $p_i$  on the size of the pie is zero, but an increase in  $p_i$  increases firm  $i$ 's share of the pie
  - ▶ so each individual has an incentive to raise its toll, battle over higher shares destroys the pie!

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- ▶ applications: console + games, operating system + software, vertical integration



# Hotelling Model of Product Differentiation

- ▶ this model allows us to study price competition as well as location (non-price) competition
- ▶ one mile long beach: 0 =left end point, 1 =right end point
- ▶  $M$  people are distributed uniformly along the beach: fraction of bathers in any section of the beach of length  $z$  is  $zM$
- ▶ location of bather  $x$  is measured relative to 0
- ▶ two ice-cream vendors are located at either end of the beach
- ▶ cost of walking:  $t$  times the distance squared

# Preferences

- ▶ utility function

$$u(x) = \begin{cases} s - p_1 - tx^2 & \text{if } x \text{ purchase from 1} \\ s - p_2 - t(1-x)^2 & \text{if } x \text{ purchase from 2} \\ 0 & \text{otherwise} \end{cases}$$

where  $s$  measures the utility for consuming an ice cream cone

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where  $s$  measures the utility for consuming an ice cream cone

- ▶ total cost of buying from vendor 1 is  $p_1 + tx^2$ , total cost of buying from vendor 2 is  $p_2 + t(1-x)^2$
- ▶ if prices are the same (and  $s$  is sufficiently large), then each bather buys from the vendor who is closer

## Local Monopoly

- ▶ no overlap in the market coverage of the two vendors when they set price equal to the monopoly price

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- ▶ no overlap in the market coverage of the two vendors when they set price equal to the monopoly price
- ▶ what is the monopoly price?
  - ▶ need to compute demand: marginal consumer  $\tilde{x}$  is indifferent between buying from vendor 1 and not buying

$$s - p_1 - t\tilde{x}^2 = 0 \Rightarrow \tilde{x} = \sqrt{(s - p_1)/t}$$

- ▶ vendor 1's demand at  $p_1$  is  $\tilde{x}$  and it chooses price to maximize

$$\pi(p_1) = p_1 \left( \sqrt{(s - p_1)/t} \right) M$$

## Local Monopoly: Solution

- ▶ differentiating and solving the FOC yields the monopoly price

$$P^M = \frac{2s}{3}$$

- ▶ need to check that  $\tilde{x}$  is indeed less than  $\frac{1}{2}$ , i.e., no overlap

$$\tilde{x}(P^M) < \frac{1}{2} \text{ iff } s < \frac{3t}{4}$$

- ▶ problem for vendor 2 is symmetric: demand is  $1 - \tilde{x}(P^M)$

## Duopoly

- ▶ together the two vendors cover the market so the marginal consumer is indifferent between buying from vendor 1 or vendor 2

$$p_1 + t\tilde{x}^2 = p_2 + t(1 - \tilde{x})^2$$
$$\Rightarrow \tilde{x} = \frac{(p_2 - p_1 + t)}{2t}$$

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- ▶ demand for vendor 1 decreases in own price and increases in rival's price
- ▶ vendor 1 chooses its price to maximize

$$\pi(p_1) = p_1 \tilde{x} M = p_1 [(p_2 - p_1 + t)/2t] M$$

- ▶ differentiating and solving for vendor 1's best reply yields

$$p_1 = \frac{p_2 + t}{2}$$



# Equilibrium

- ▶ solving vendor 2's problem yields

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- ▶ intersection of the best replies

$$p_1 = p_2 = t$$

- ▶ marginal consumer is  $\tilde{x} = 1/2$
- ▶ equilibrium profits are  $\pi_1 = \pi_2 = \frac{t}{2}M$

## Location Choice

- ▶ we can endogenize the location of the vendors using a two-stage game: vendors first choose their location and then compete in prices
- ▶ question: will vendors locate as far from each other as possible (maximal differentiation) or as close as possible (minimal differentiation)?

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- ▶ question: will vendors locate as far from each other as possible (maximal differentiation) or as close as possible (minimal differentiation)?
- ▶ assume vendor 1 is located at point  $a$  and vendor 2 is located at  $1 - b$  where  $1 - b \geq a$ 
  - ▶  $a + b = 1$  is minimal differentiation
  - ▶  $a + b = 0$  is maximal differentiation

## Location Choice

- ▶ stage 2: given any pair of locations  $(a, 1 - b)$ , demand functions (derived from utility maximization) are

$$D_1(p_1, p_2) = \tilde{x} = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}$$

$$D_2(p_1, p_2) = 1 - \tilde{x} = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)}$$

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- ▶ Nash equilibrium prices given locations are

$$p_1^*(a, b) = t(1 - a - b) \left( 1 + \frac{a - b}{3} \right)$$

$$p_2^*(a, b) = t(1 - a - b) \left( 1 + \frac{b - a}{3} \right)$$

## Location Choice (Cont.)

- ▶ substituting the equilibrium prices into the vendors' profit functions yields the reduced form profit functions for  $i = 1, 2$

$$\pi_i(a, b) = p_i^*(a, b) D_i(a, b, p_1^*(a, b), p_2^*(a, b))$$

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- ▶ given  $b$ , firm 1 chooses  $a$  to maximize profits

$$\begin{aligned} \frac{\partial \pi_1(a, b)}{\partial a} &= \frac{\partial p_1^*}{\partial a} D_1 + p_1^* \left[ \frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_1} \frac{\partial p_1^*}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} \right] \\ &= p_1^* \left[ \frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} \right] \end{aligned}$$

where the last equality follows the envelope theorem:

$$\frac{\partial \pi_1(a, b)}{\partial p_1} = D_1 + p_1^* \frac{\partial D_1}{\partial p_1} = 0$$



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- ▶ verify:  $\frac{\partial D_1}{\partial a} = \frac{3-5a-b}{6(1-a-b)}$  and  $\frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} = \frac{a-2}{3(1-a-b)}$

## Location Choice (Cont.)

- ▶ combining the expressions, we obtain

$$\frac{\partial \pi_1(a, b)}{\partial a} < 0$$

- ▶ implication: vendor 1 wants to locate as far from vendor 2 as possible, the same is true for vendor 2
  - ▶ subgame perfect equilibrium is  $a^* = b^* = 0$  and  $p_1^* = p_2^* = t$
  - ▶ social optimal:  $a = b = 1/4$ , thus, free entry results in too much differentiation

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  - ▶ social optimal:  $a = b = 1/4$ , thus, free entry results in too much differentiation
- ▶ basic conflict: locate where demand is ( $\frac{\partial D_1}{\partial a} > 0$ , move to the center)  $\leftrightarrow$  stay away from price competition ( $\frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} < 0$ , go to the ends)
  - ▶ depends on travel costs: quadratic travel costs  $\rightarrow$  the second force dominates

# Salop's Circular City

- ▶ consumers are distributed uniformly on a circle with circumference 1
- ▶ each consumer demands one unit, willingness to pay is  $s$
- ▶ transport costs are linear:  $t$  per unit of distance traveled
- ▶  $N$  firms are symmetrically located around the circle at distance  $\frac{1}{N}$  apart: ignore the location choice problem
- ▶ entry cost is  $f$ , production cost is  $c$  per unit

# Demand

- ▶ assumption:  $t$  is sufficiently large that a symmetric pure strategy equilibrium exists in which firms service the neighborhoods around their locations
  - ▶ each firm  $i$  competes only with firms  $i - 1$  and  $i + 1$
  - ▶ assume these firms set same price  $p$

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$$p_i + tx = p + t \left( \frac{1}{n} - x \right)$$

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$$p_i + tx = p + t \left( \frac{1}{n} - x \right)$$

- ▶ thus demand for firm  $i$  is

$$D(p_i, p) = 2x = t^{-1} \left( p + \frac{t}{n} - p_i \right)$$

# Equilibrium

- ▶ firm  $i$  chooses  $p_i$  to maximize

$$\pi_i(p_i, p) = (p_i - c) t^{-1} \left( p + \frac{t}{n} - p_i \right)$$



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- ▶ imposing symmetry yields the symmetric equilibrium price and profit

$$p^* = \frac{t}{n} + c, \quad \pi^* = \frac{t}{n^2}$$

- ▶ equilibrium number of firms: zero profit condition

$$\pi^* - f = 0 \Rightarrow n^* = \sqrt{\frac{t}{f}}$$

## Does the market generate too much or too little variety?

- ▶ social planner's problem: choose  $n$  to minimize sum of fixed costs and consumer's transport cost

$$\min_n \left\{ nf + t(2n) \int_0^{\frac{1}{2n}} x dx \right\} = \min_n \left\{ nf + \frac{t}{4n} \right\}$$

- ▶ remark: as long as market is covered, total surplus is fixed at  $(s - c)$

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- ▶ solution

$$\tilde{n} = \sqrt{\frac{t}{4f}} = \frac{n^*}{2}$$

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- ▶ remark: as long as market is covered, total surplus is fixed at  $(s - c)$
- ▶ solution

$$\tilde{n} = \sqrt{\frac{t}{4f}} = \frac{n^*}{2}$$

- ▶ conclusion: market generates too much product variety
  - ▶ intuition: entrants do not take into account the impact of their entry on profits of incumbents
  - ▶ equi-distant spacing of products can be justified if transport costs are quadratic

## Dixit-Stiglitz-Spence Model

- ▶ consumers have CES preferences over a numeraire good  $q_0$  and  $n$  differentiated goods

$$U \left( q_0, \left( \sum_{i=1}^n q_i^\sigma \right)^{1/\sigma} \right), \sigma \leq 1$$

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- ▶ maximize utility subject to budget constraint

$$q_0 + \sum_{i=1}^n p_i q_i = I$$

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$$q_0 + \sum_{i=1}^n p_i q_i = I$$

- ▶ each firm produces one good, production involves a fixed cost  $f$  and marginal cost  $c$



## Demand

- ▶ substituting for  $q_0$  using the budget constraint, demands for the differentiated products are given by the solution to the system of  $n$  equations

$$\frac{\partial U}{\partial q_0} p_i - \frac{\partial U}{\partial q_i} \left( \sum_{i=1}^n q_i^\sigma \right)^{\frac{1}{\sigma}-1} q_i^{\sigma-1} = 0, \quad i = 1, \dots, n$$

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- ▶ each good competes with every other good, suppose  $n$  is sufficiently large

$$q_i \approx k p_i^{-\frac{1}{1-\sigma}}$$

where  $k \equiv \frac{\partial U}{\partial q_0} / \frac{\partial U}{\partial q_i} \left( \sum_{i=1}^n q_i^\sigma \right)^{\frac{1}{\sigma}-1}$  is approximately a constant

# Supply

- ▶ firm  $i$  chooses  $p_i$  to solve

$$\pi_i(p_i) = (p_i - c) q_i(p_i) - f$$

- ▶ differentiating and solving

$$(p_i - c) \left( -\frac{1}{1 - \sigma} \right) k p_i^{-\frac{1}{1 - \sigma} - 1} + k p_i^{-\frac{1}{1 - \sigma}} = 0$$

$$\Rightarrow p_i = \frac{c}{\sigma}$$

- ▶ equilibrium prices are independent of  $n$

# Equilibrium

- ▶ to determine  $n^*$ ,  $q^*$ , we can use zero-profit condition and FOC

$$\left(\frac{c}{\sigma} - c\right) q^* = f$$

$$\frac{\partial U}{\partial q_0} p_i = \frac{\partial U}{\partial q_i} n^{\frac{1}{\sigma}-1}$$

- ▶ solution to these two equations turns out to be the solution to the social planner's problem