

## Agenda

- price dispersion and search
  - theory: Varian (1980)
  - empirical: Sorensen (2000)
- structural estimation of search models
  - Hong and Shum (2006), Hortacsu and Syverson (2004), De Los Santos, Hortacsu, and Wildenbeest (2012)

## Search

- an important assumption of the models we have developed in the previous lectures is that consumers know the prices that sellers are charging
- however, in most retail markets, consumers do not know the prices charged by different retailers and have to learn them
  - this activity is costly, so consumers are unlikely to be fully informed
  - one of the main uses of the Internet is to provide price information cheaply
- question: how does the consumer's lack of information about prices affect competition among firms?

## A Simple Model: Varian (1980)

- supply
  - $N$  firms supply a homogenous good
  - product costs are zero
- demand
  - consumers have unit demands and willingness to pay of  $r$
  - two types of consumers:  $I$  informed and  $M$  uninformed consumers
    - \* an informed consumer knows the prices charged by all  $N$  firms
    - \* an uninformed consumer does not know the prices and randomly selects one firm to shop

## The Game

- $n$  firms post prices simultaneously
  - informed consumers buy from the firms setting the lowest price if it does not exceed  $r$
  - uninformed consumers randomly select a firm and buy if the firm's price does not exceed  $r$
- let  $U = \frac{M}{n}$  be the number of uninformed consumers each firm gets
- payoffs

$$\pi_i(p_i, p_{-i}) = \begin{cases} p_i(U + I) & \text{if } p_i < p_j, j \neq i \\ p_i(U + \frac{I}{m}) & \text{if } p_i = p_j, j = 1, \dots, m-1 \\ p_i U & \text{if } r \geq p_i > p_j, j \neq i \end{cases}$$

## Equilibrium Analysis

- claim 1: there is no equilibrium in which all firms charge the same price
- proof: without loss of generality, assume that  $n = 2$ 
  - $p = 0$  is not an equilibrium because each firm can raise price to  $r$  and earn  $rU$
  - if they charge  $r \geq p > 0$ , then a slight cut in price gets all the informed consumers, which increases profits.
- thus, the Law of One Price does not hold!

## Equilibrium Analysis (Cont.)

- claim 2: there is no equilibrium in pure strategies
- proof: by previous claim, one firm sets a higher price than the other, let  $p^L < p^H$ 
  - clearly,  $p^H = r$  since firm  $H$ 's profits are  $pU$  which is strictly increasing in  $p$
  - but then firm  $L$  should raise price  $p^L$  until it is slightly below  $r$
  - but then firm  $H$  wants to undercut  $p^L$
- we need to look for an equilibrium in which firms cannot forecast the prices of their rivals and undercut them
  - firms are perceived as choosing prices randomly

### Mixed-Strategy

- let  $F(p)$  denote the probability that a rival posts a price less than  $p$ , then the expected profit to a firm when it charges  $p$  is

$$\pi(p) = [1 - F(p)]^{n-1} p(U + I) + \left\{ 1 - [1 - F(p)]^{n-1} \right\} pU$$

- the first term on the RHS is the probability that  $p$  is the lowest price, in which case firm demand is  $U + I$
- the second term on the RHS is the probability that  $p$  is not the lowest price, in which case firm demand is only  $U$
- in a mixed strategy equilibrium, the firm's profits at every price  $p$  has to be constant:  $\pi(p) = k$ 
  - solving the equilibrium equation yields

$$1 - F(p) = \left[ \frac{k - pU}{pI} \right]^{\frac{1}{n-1}}$$

### Mixed-Strategy (Cont.)

- clearly, the upper bound on prices is  $r$ , therefore, the unknown constant  $k$  satisfies

$$F(r) = 1 \Rightarrow k = rU$$

- highest price, demand is only  $U$
- substituting, we obtain

$$1 - F(p) = \left[ \frac{(r - p)U}{pI} \right]^{\frac{1}{n-1}}$$

- the lower bound of the set of prices that firms will charge is obtained by setting  $F(p) = 0$

$$\underline{p} = \frac{rU}{U + I}$$

### Mixed-Strategy (Cont.)

- the lower bound  $\underline{p}$  is strictly positive
  - firms earn positive profits: escape the Bertrand trap
  - the equilibrium density of prices is U-shaped: firms will tend to either price near  $r$  or near the lower bound
- intuition: firm either goes for the informed consumers or is content exploiting the uninformed consumers
- remarks: as  $U$  falls, market becomes more competitive, prices fall and lower prices more likely

### Search Cost: Internet

- prior to the Internet, search costs were attributed to dispersion in the geographical location of stores
  - it was costly to visit every store and determine who was offering the lowest price
- however, price search engines like *Pricewatch.com* provide consumers with lost of prices at very low cost
  - online travel agents like *Travelocity* and *Expedia*
- this led many researchers to predict less price dispersion and lower margins in online markets than in brick and mortar markets

### Search Cost: Internet (Cont.)

- it is much easier to test this prediction in online markets
  - even if products are identical (e.g., “Da Vinci Code” book) when they are sold in brick and mortar stores, they are differentiated by location
  - products in online markets are not differentiated by location
- nevertheless, a long list of papers have found that price dispersion among E-retailers is similar to that of brick and mortar retailers and margins are not extremely low (e.g., Amazon reports average markups of 15%)
  - one reason may be obfuscation, online retailers try to make it difficult for consumers to determine the true price, e.g., shipping cost, taxes, etc.

### Sorensen (2000)

- objective: test a comparative static prediction of Varian’s theory of price dispersion
  - consumers have stronger incentives to search for lowest price when they have to purchase drug frequently
- hypothesis: lower markups, less dispersion for drugs that are purchased more frequently
- data: prices of 152 top-selling prescription drugs
  - 10 pharmacies in Middletown, NY
  - 11 pharmacies in Newburgh, NY

## Data Overview

- in New York, pharmacies are required to post prices on a large poster provided by State Board of Pharmacy
- the two towns are geographically isolated so consumers in these towns have to buy drugs at these pharmacies
- drug characteristics were collected from Moseby's Genrx: primary use, dosage, duration of therapy, wholesale prices
- drug dosage/therapy duration used to compute expected number of purchase times per year

## Descriptive Statistics

- price variation: posted prices averaged \$13.17, with 10th and 90th percentiles of \$4.91 and \$25.36
- pharmacies could not be easily sorted into low-price and high-price categories

PRICE RANKINGS BY PHARMACY  
A. MIDDLETOWN

PHARMACY	PRICE GROUP		
	Lowest 3	Middle 4	Highest 3
Eckerd	45	103	10
Eckerd	29	102	27
Immediate	43	54	61
K-Mart	56	57	45
Medicine Shoppe	99	49	10
Price Chopper	80	67	11
Rite-Aid	3	11	144
Rite-Aid	2	18	138
Rx Place	38	104	16
Wal-Mart	79	67	12

## Descriptive Statistics(Cont.)

B. NEWBURGH

PHARMACY	PRICE GROUP		
	Lowest 3	Middle 3	Highest 3
Ace	26	112	30
Hudson	33	106	29
Medical Arts	73	65	30
Price Chopper	134	27	7
Rite-Aid	4	23	141
Rite-Aid	10	45	113
Rite-Aid	18	34	116
Rx Place	64	70	34
Wal-Mart	142	22	4

- (kind of) consistent with randomization on individual drug prices (a lot of uncertainty in rivals' prices)

## Regression I

- regression equation

$$RANGE_{ij} = \beta_0 + \beta_1 PFREQ_i + \text{other controls} + \epsilon_{ij}$$

where  $RANGE_{ij}$  is price range of drug  $i$  in city  $j$ ,  $PFREQ_i$  is purchase frequency of drug  $i$

PRICE DISPERSION AND PURCHASE FREQUENCY				
	DISPERSION MEASURE			
	Range (1)	Standard Deviation (2)	Residual Range (3)	Residual Standard Deviation (4)
Purchase frequency	-.336 (.123)	-.173 (.076)	-.266 (.061)	-.102 (.016)
Wholesale cost	.280 (.033)	.180 (.020)	.215 (.043)	.069 (.014)
Branded with generic competition	-.803 (1.037)	-1.480 (.641)	-1.842 (.861)	-.362 (.248)
Branded without ge- neric competition	-1.505 (2.108)	-2.010 (1.303)	-1.967 (1.060)	-.772 (.339)
Newburgh dummy	-2.686 (.633)	-3.172 (.314)	-1.493 (.791)	-.916 (.271)
Constant	20.070 (4.343)	7.321 (2.563)	14.570 (1.062)	5.283 (.448)
$R^2$	.371	.447	.258	.253
$\hat{\rho}$	.338	.585	.149	.648

## Regression II

AVERAGE MARGINS AND PURCHASE FREQUENCY			
	DEPENDENT VARIABLE		
	Average Margin (1)	Average Price (2)	Average Relative Margin (3)
Purchase frequency	-.262 (.102)	-.137 (.105)	.001 (.003)
Wholesale cost	...	.994 (.032)	...
Wholesale cost × generic dummy	...	-.208 (.059)	...
Branded with generic competition	2.101 (.720)	-.668 (1.056)	-.235 (.020)
Branded without generic competition	3.415 (1.660)	-.123 (1.891)	-.255 (.046)
Newburgh dummy	1.681 (.174)	1.648 (.140)	.047 (.005)
Constant	12.69 (2.435)	11.86 (2.581)	.463 (.068)
$R^2$	.229	.895	.510
$\hat{\rho}$	.915	.936	.898

- margins are negatively correlated with drug frequency: 37% lower for drugs that are purchased monthly versus drugs purchased only once

## Remarks

- alternative explanations
  - pharmacy heterogeneity: fixed effects account for 33% of the dispersion in prices
  - cost heterogeneity: differences in drug acquisition costs across pharmacies are too small
- main conclusion: price dispersion is substantial, it is positively correlated with drug purchase frequency

## Consumer Search Models

- Varian (1980) isn't technically a model of consumer search
- various subsequent authors proposed models in which consumers' "informedness" is endogenous
  - consumers have search costs
  - the search for lower prices if expected benefit of search is greater than search cost
- two main modeling approaches
  - fixed sample size search
  - sequential search

## Nonsequential Search: Burdett and Judd (1983)

- consumers know price distribution  $F(p)$
- choose in advance how many price quotes to obtain
  - one rational: maybe price quotes come after a delay
- if cost of each price quote is  $c$ , then getting  $n$  price quotes gives an expected total purchase cost equal to

$$cn + \int_0^{\infty} np [1 - F(p)]^{n-1} dF(p)$$

- this is a convex function of  $n$ , so there exists a unique integer that minimizes total purchase cost (or two adjacent integers that tie)

### Sequential Search: Stahl (1989)

- consumers know price distribution  $F(p)$ 
  - fraction  $\mu$  of zero-search-cost “shoppers”
  - fraction  $1 - \mu$  have common search cost  $c > 0$  per price quote
- if best price found so far is  $z$  and  $z < WTP$ , expected benefit from an additional search is

$$\int_{\underline{p}}^z (z - p) dF(p)$$

- search again if expected benefit is greater than cost  $c$
- optimal search rule is a reservation price rule: if you find a price smaller than  $r^*$ , then buy; otherwise searching

### Remarks

- most models assume consumers know the underlying price distribution
- what if instead they learn about the distribution through search?
- Rothschild (1974)
  - many properties of optimal search behavior look similar
  - but no longer possible to characterize equilibrium price distribution

### Structural Estimation

- some papers test the predictions of search models, e.g., Sorensen (2000)
- but what about estimating search models directly?
- can we use price distribution to estimate search costs?
  - Hong and Shum (2006), Hortacsu and Syverson (2004), etc.

### Hong and Shum (2000): Nonsequential Search Case

- idea: if we impose the equilibrium restrictions of a search model, we can recover the unobserved distribution of search cost from the observed distribution of prices
- cutoff-points: expected price savings from the  $k^{th}$  price quote

$$\Delta_k = E[\min(p_1, \dots, p_k)] - E[\min(p_1, \dots, p_{k-1})]$$

- consumers who obtain  $k$  price quotes must have search costs between  $\Delta_k$  and  $\Delta_{k+1}$



### Hong and Shum (2000): Nonsequential Search Case (Cont.)

- how many consumers are obtaining  $k$  price quotes? use supply-side equilibrium condition: expected profits must equal at all prices

$$(\bar{p} - c) \hat{q}_1 = (p_i - c) \left[ \sum_{k=1}^K \hat{q}_k k \left(1 - \hat{F}_p(p_i)\right)^{k-1} \right]$$

- since  $\sum_{k=1}^K \hat{q}_k = 1$ , above gives  $n - 1$  equations to solve for  $K$  unknowns  $\{c, \hat{q}_1, \dots, \hat{q}_{K-1}\}$
- use estimates  $\hat{q}_k$  to solve for  $F_c(\Delta_1), \dots, F_c(\Delta_{K-1})$

### Hortacsu and Syverson (2004)

- question: what explains the diffuse prices of seemingly similar mutual funds? specifically, what level of search costs would rationalize the observed price dispersion?
- strategy: use price and quantity data from S&P 500 funds, estimate a search model that allows for vertical differentiation in addition to search frictions

### Model: Brief Overview

- consumers know empirical CDF of offered utilities  $u$
- optimal search implies cutoff points of the search cost distribution

$$c_j = \sum_{k=j}^N \rho_k (u_k - u_j)$$

where  $\rho_k$  is the sampling probability of  $k$

- market shares can be mapped to these cutoffs

$$\begin{aligned} q_1 &= \rho_1 [1 - G(c_1)] \\ q_2 &= \rho_2 [1 - G(c_1)] + \frac{\rho_2}{1 - \rho_1} [G(c_1) - G(c_2)] \\ &\vdots \end{aligned}$$

- search cost distribution: recovered from market shares and optimal search condition

### De Los Santos, Hortacsu, and Wildenbeest (2012)

- question: which model of search, sequential or non-sequential, is better supported by data on actual consumers' searches
- strategy: use Comscore data on book purchases (and searches) to test key predictions of the sequential model: no recall (should always buy from the last store visited), and price dependence (decision to search again should depend on price of last store searched)

## Comscore Data

- tracks web usage of a large sample of users, including online purchases
- not a random sample of internet users (because users must agree to be tracked)
- this paper: focus on searches and purchases of books
  - approx 15,500 purchase transactions from 2002 to 2004
  - approx 325,000 site visits to 15 online bookstores

## Testing Predictions of Sequential Search

- no recall: under sequential search, buyers should purchase from last store visited (unless their search exhausts all stores)
- price dependence: under sequential search, the probability of another search depends on the value of the last price quote (only keep searching if last quoted price is high)

## Summary Stats: Stores

Bookstore	Transactions		Visits	
	Number	%	Number	%
Amazon	10,197	65.5%	249,593	76.3%
Barnes and Noble	3,042	19.6%	25,758	7.9%
<i>Book Clubs</i>				
christianbook.com	615	3.9%	3,968	1.2%
doubledaybookclub.com	468	3.0%	4,001	1.2%
eharlequin.com	61	0.4%	3,647	1.1%
literaryguild.com	322	2.1%	3,500	1.1%
mysteryguild.com	187	1.2%	2,095	0.6%
<i>Other Bookstore</i>				
1bookstreet.com	10	0.1%	120	0.0%
allbooks4less.com	5	0.0%	199	0.1%
alldirect.com	27	0.2%	490	0.1%
ecampus.com	114	0.7%	1,206	0.4%
powells.com	68	0.4%	1,326	0.4%
varsitybooks.com	16	0.1%	218	0.1%
walmart.com	183	1.2%	28,663	8.8%
booksamillion.com	246	1.6%	2,290	0.7%
Total	15,561	100.0%	327,074	100.0%

## Summary Stats: Searches

	2002		2004	
	Mean	Std. Dev.	Mean	Std. Dev.
<i>Duration of each website visit (in minutes)</i>				
Visits not within 7 days of transaction	8.89	13.03	7.69	12.36
Visits within 7 days, excluding transactions	12.72	15.83	11.02	15.00
Visits within 7 days, including transactions	19.04	18.26	15.74	17.37
Transactions only	28.06	17.69	26.08	17.71
Total duration, excluding transaction visits	32.47	49.80	38.41	78.33
Total duration, including transaction visits	43.88	43.27	47.43	66.11
Number of stores searched	1.27	0.54	1.30	0.56
Number of books per transaction	2.38	2.10	2.20	1.95
Transaction expenditures (books only)	36.67	40.64	32.21	35.68
Number of books purchased	17,956		17,631	
Number of transaction sessions	7,559		8,002	
Number of visits within 7 days	18,350		25,556	
Number of visits not within 7 days	94,011		189,157	

## Evidence of Recall

Search window	No. of stores visited		If 2 or more stores, bought from:		Exhausted search?
7 Days	One	76%	Last store sampled Recalled	65% 35%	55%
	2 or more	24%			
6 Days	One	77%	Last store sampled Recalled	64% 36%	55%
	2 or more	23%			
5 Days	One	79%	Last store sampled Recalled	63% 37%	55%
	2 or more	21%			
4 Days	One	80%	Last store sampled Recalled	61% 39%	55%
	2 or more	20%			
3 Days	One	82%	Last store sampled Recalled	61% 39%	56%
	2 or more	18%			
2 Days	One	84%	Last store sampled Recalled	61% 39%	56%
	2 or more	16%			
1 Day	One	86%	Last store sampled Recalled	61% 39%	56%
	2 or more	14%			
Same day	One	90%	Last store sampled Recalled	62% 38%	58%
	2 or more	10%			

## Evidence of Price Dependence

- dependent variable: indicator for searching more than 1 store

Variable	(A)	(B)	(C)	(D)	(E)
<i>Panel A. All transactions</i>					
<i>Coefficients</i>					
Intercept	-1.817 (0.093)	-1.796 (0.123)	-0.818 (0.154)	0.295 (0.133)	—
First price lower or equal	-0.071 (0.119)	0.090 (0.152)	0.040 (0.157)	-0.223 (0.165)	-0.073 (0.371)
Loyal	—	—	-1.446 (0.151)	—	—
<i>Average marginal effects</i>					
First price lower or equal	-0.008 (0.014)	0.011 (0.019)	0.005 (0.019)	-0.055 (0.041)	-0.015 (0.078)
Loyal	—	—	-0.171 (0.017)	—	—
Number of observations	2,593	1,504	1,504	649	235

- weak relation between decision to continue searching and observed prices

### Outline of Demand Estimation with Search

- basic idea: probability  $i$  purchases from  $j$  is  $P_{iS}P_{ij|S}$ 
  - $P_{ij|S}$  is the probability of choosing  $j$  if set of searched stores was  $S$
  - $P_{iS}$  is probability  $i$  chooses to search the set of stores  $S$
- expected payoff if search set is  $S$

$$m_{iS} = E \left[ \max_{j \in S} \{ \mu_j + X_j \beta_j + \alpha_i p_j + \epsilon_{ij} \} \right] - kc_i$$

### Computing $P_{iS}$

- if consumers know  $\epsilon$  and prices have T1EV distribution, then

$$E \left[ \max_{j \in S} \{ u_{ij} \} \right] = \alpha_i \sigma \log \left( \sum_{j \in S} \exp \left[ \frac{\mu_j + X_j \beta_j + \epsilon_{ij} + \alpha_i \gamma_j}{\alpha_i \sigma} \right] \right)$$

where  $\gamma_j$  and  $\sigma$  are location and scale parameters of stores' price distributions

- to smooth the choice probabilities, add a logit shock  $\varsigma_{iS}$  to each  $m_{iS}$ , so

$$P_{iS} = \frac{\exp(m_{iS}/\sigma_\varsigma)}{\sum_{S'} \exp(m_{iS'}/\sigma_\varsigma)}$$

### Estimation Details

- $P_{ij|S}$  cannot be calculated analytically
- calculate  $\gamma_j$ 's and  $\sigma_p$  from observed price distributions in a first step, then treat as known by consumers
- normalization: variance of  $\epsilon$ 's set to 1, so variance of choice-set error  $\sigma_\varsigma$  is estimated relative to variance of store-specific errors

## Main Results

- average search costs around \$1.35
- own-price elasticities around  $-1$  for Amazon, around  $-2$  for B&N
- if demand is estimated assuming full information, then estimated elasticities are much smaller
  - consumers' unresponsiveness to price differences attributed to small  $\alpha$  instead of to search frictions