Agenda

- demand for differentiated products: Generation III models
- Bresnahan (1987)
- BLP (1995)

Demand for Differentiated Products: Generation III Models

- our basic problem is that choices are correlated
  - if a consumer chooses product j, it means that she values the characteristics of product j, therefore, when p_j increases, she will tend to choose a product with similar characteristics
- the correlation in individual choices occurs through the idiosyncratic term ε_{ij}: in the previous models, these shocks were assumed to be i.i.d
  - why not assume that they are correlated across products and estimate the variance-covariance matrix of ε_{ij}?
- the problem is that this approach simply reintroduces the dimensionality problem: have $\frac{J^2}{2}$ parameters to estimate

Random Coefficients

- an alternative approach is to impose structure on the form of correlation in idiosyncratic shocks: the idea is to specify the correlation as a linear function of the product characteristics
- consumer i's utility for product j

$$u_{ij} = x_j \beta_i - \alpha p_j + \xi_j + \epsilon_{ij}$$

  where $\beta_{ik} = \beta_k + \sigma_k \varsigma_{ik},\ k = 1, ..., K$

- thus,

$$u_{ij} = x_j \beta - \alpha p_j + \xi_j + \nu_{ij}$$

  where $\nu_{ij} = \sum_{k=1}^{K} x_{jk} \sigma_k \varsigma_{ik} + \epsilon_{ij}$

  - ϵ_{ij} is i.i.d. Type I extreme value, ς_{ik} is standard normal

Comments

- the idiosyncratic shock $\nu_{ij} = \sum_{k=1}^{K} x_{jk}\sigma_k\varsigma_{ik} + \epsilon_{ij}$ is correlated across products
  - if consumer i has a high realization of ς_{ik} for characteristic k, then she values this characteristic in all J products
  - consequently, if p_j increases, she will tend to switch to a product that has a lot of x_k
- in modeling the correlation in this way, we have added K parameters to the model, one for each characteristic
- the variation that identifies $\sigma = (\sigma_1, ..., \sigma_K)$ are changes in prices or products that generate substitution patterns that differ from those predicted by the logit model
  - if the data-generating model is logit, then we will estimate σ to be zero (i.e., the distributions of β_i is degenerate at β)

Estimation Algorithm

- integrate over ν_{ij} to obtain market shares

$$s_j(\delta, \theta) = \int \frac{\exp\left(\delta_j + \sum_{k=1}^{K} x_{jk}\sigma_k\varsigma_{ik}\right)}{1 + \sum_{m=1}^{J} \exp\left(\delta_m + \sum_{k=1}^{K} x_{mk}\sigma_k\varsigma_{ik}\right)} d\Phi(\varsigma)$$

  - this is K-dimension integral which cannot be computed easily
- Monte Carlo simulation: draw $\{\varsigma_{ik}^r\}_{r=1}^{R}$ from $\Phi(\cdot)$ and take the average

$$s_j(\delta, \theta) = \frac{1}{R}\sum_{r=1}^{R} \frac{\exp\left(\delta_j + \sum_{k=1}^{K} x_{jk}\sigma_k\varsigma_{ik}^r\right)}{1 + \sum_{m=1}^{J} \exp\left(\delta_m + \sum_{k=1}^{K} x_{mk}\sigma_k\varsigma_{ik}^r\right)}$$

- equate actual to simulated market shares and invert the system to obtain the mean utilities, or equivalently $\xi(\theta, s)$, then interact $\xi(\theta, s)$ with instruments z and find the value of θ that makes the sample moments as close to 0 as possible

Remarks

- the simulated market shares enter non-linearly in the moment conditions so nice properties of simulation estimators are not valid
- IIA property continues to hold at the individual level: ratio of choice probabilities does not depend upon number or utility of the other alternatives
- but, market shares no longer have the IIA property, aggregating over the realizations of ς implies that ratio of market shares depends upon the number and characteristics of alternative products

Data on Consumer Characteristics

- in many cases, we can observe the distribution of consumer characteristics
  - e.g., from census data, we can learn the income, education, age, and family size of consumers living in a particular geographical market
- let z_i denote the vector of observable consumer characteristics, then our model of how consumer preferences vary as a function of observed and unobserved individual characteristics is that

$$\nu_{ij} = \sum_{k=1}^{K} x_{jk} \left(\pi_k z_{ik} + \sigma_k \varsigma_{ik} \right) + \epsilon_{ij}$$

- the choice probabilities for consumer i are obtained by integrating over the idiosyncratic shock ϵ as above

Estimation Algorithm

- to obtain the market share of product j, we need to integrate over
  - the unobserved characteristics ς which are distributed as standard normal
  - the observed characteristics which are distributed in the population according to some joint distribution G, this distribution is obtained (parametrically or non-parametrically) from census data
- estimation
  - draw vectors of consumer characteristics from these distributions, determine individual choices
  - aggregate to obtain predicted market shares
  - solve demand system to obtain $\xi(\theta, s)$ and then interact with instruments (x, w) to do GMM

Remarks

- the demographic information reduces the reliance on parametric assumptions about the distribution of consumer heterogeneity
- it also allows the model to incorporate differences in the distribution of consumers across markets and their impact on aggregate demand
  - for example, all empirical evidence suggests that the impact of price on consumer demand depends on the consumer's income
  - so if the distribution of income varies across geographical market, then each market has a different price coefficient
  - the random coefficients model with demographic characteristics captures this interaction

Remarks (Cont.)

- it provides an approximation to the demand surface that is tailored to each market and does not impose one approximation to all markets
  - better fit leads to more precise parameter estimates
  - provides a tool for making predictions of likely outcomes in new markets or from policies that would affect the distribution of consumer characteristics

Pricing Equations

- suppose there are N firms in the market, indexed by t
  - firms may produce more than one product, let J_t denote the number of products by firm t
  - firms choose prices, let p_t denote the price vector for firm t and p_{-t} the prices of its rivals
- each firm t choose p_t to maximize

$$\pi_t(p_t, p_{-t}) = \sum_{j \in J_t} [p_j - mc_j] M s_j(x, p, \xi)$$

FOC

- first-order equations for product j is

$$s_j(x, p, \xi) + \sum_{r \in J_t} (p_r - mc_r) M \frac{\partial s_r}{\partial p_j} = 0$$

- in matrix notation

$$s + (p - mc)\Delta = 0$$

  where Δ_{ij} is nonzero for the elements of a row that are produced by the same firm as the row good (diagonal if each firm produces only one good)

Estimation Equation

- to complete the model, we need to specify a functional form for marginal cost

$$\log(mc_j) = w_j \gamma + \omega_j$$

- substituting back, we obtain the pricing equations

$$\log\left(p - \Delta^{-1} s\right) - w\gamma = \omega(\theta)$$

- note that Δ is the derivative of market demand so it depends on the demand parameters
  - the pricing and demand equations can be estimated jointly using simulated method of moment estimator

Remarks

- in practice, the inclusion of the pricing equations really helps identify the demand parameters of the random coefficient model
  - the demand model is often too flexible for the data: not enough variation across products and markets relative to the approximations
- in some cases, the authors does not estimate product marginal costs but back them out estimates from FOC directly

Application: Bresnahan (1987)

TABLE I

Year	(1) Auto Production[a]	(2) Real Auto Price-CPI[b]	(3) % Change Auto Price-Cagan[c]	(4) Auto Sales[d]	(5) Auto Quantity Index[e]
1953	6.13	1.01	NA	14.5	86.8
1954	5.51	0.99	NA	13.9	84.9
1955	7.94	0.95	−2.5	18.4	117.2
1956	5.80	0.97	6.3	15.7	97.9
1957	6.12	0.98	6.1	16.2	100.0

Notes: [a] Millions of units over the model year. [Source: *Automotive News*.]
[b] (CPI New automobile component)/CPI. [Source: *Handbook of Labor Statistics*.]
[c] Adjusted for quality change. [See Cagan (1971), especially pp. 232–3.]
[d] Auto output in constant dollars, *QIV* of previous year through *QIII* of named year, in billions of 1957 dollars. [Source: *National Income and Product Accounts*.]
[e] (4)/(2), normalized so 1957 = 100.

- in 1955, US auto production was 45% higher than in 1954 and 1956, quality adjusted prices were substantially lower
- one year shift towards smaller, lower value cars
- question: what is the explanation?

Demand Side Shocks?

TABLE II

Year	(6) Per Capita Disposable Personal Income[f]	(7) Interest Rate[g]	(8) Durables Expenditures (Non-Auto)[h]	(9) Automakers Profits[i]
1953	1623	1.9	14.5	2.58
1954	1609	0.9	14.5	2.25
1955	1659	1.7	16.1	3.91
1956	1717	2.6	17.1	2.21
1957	1732	3.2	17.0	2.38

Notes: [f] Billions of 1957 dollars, *QIV* of previous year through *QIII* of named year. [Source: *National Income and Product Accounts*.]
[g] Three-month T-bill rate. [Source: *Statistical Abstract*.]
[h] Durables component of consumer expenditures minus component for automobiles and parts, billions of 1957 dollars. [Source: *National Income and Product Accounts*.]

- 1955 was a year of mild expansion: income rose, interest rates increase, as did sales of durables other than autos
- expansion continued in 1956 and sales of non-auto durables continued to increase, in contrast to autos

Competition: Less or More?

- accounting profits of automakers increased but this is misleading due to the way fixed costs, which are large, are treated
  - accounting practice spreads these costs out smoothly over many years
  - high unit sales years, like 1955, tend to be "profitable" in the accounting sense but not in the economic, price-cost margins sense
- Bresnahan's Hypothesis: automakers had a price war in 1955, colluded in 1954 and 1956
- to test this hypothesis, we need to estimate markups

Demand Model

- N products are ordered in quality x
- utility function
  - if buy car of quality x, $U(x, v, y) = vx + y - p$
  - otherwise, $U(x, v, y) = v\gamma + y - E$ (should normalize γ to zero)
- v is distributed uniformly with density δ on $[0, V_{max}]$

Demand Model (Cont.)

- ordering the products from lowest to highest, the model yields the following demands

$$q_1 = \delta\left[\frac{P_2 - P_1}{x_2 - x_1} - \frac{P_1 - E}{x_1 - \gamma}\right]$$
$$q_i = \delta\left[\frac{P_j - P_i}{x_j - x_i} - \frac{P_i - P_h}{x_i - x_h}\right]$$
$$q_n = \delta\left[v_{max} - \frac{P_n - P_{n-1}}{x_n - x_{n-1}}\right]$$

- product quality depends upon the physical characteristics of the cars

$$x = \sqrt{\beta_0 + \sum_k z_k \beta_k}$$

Supply Side

- fixed costs are increasing in quality, product qualities are fixed prior to the pricing stage and fixed costs are sunk
- marginal costs are increasing and convex in quality

$$C(q,x) = A(x) + mc(x)\,q$$

where $mc(x) = \mu e^x$

  - firms choose product prices

FOC

- in the vertically differentiated product space, firms only compete against their neighbors
- if firms producing products i and $i+1$ compete, then the first order conditions are

$$q_i + [P_i - mc(x_i)]\frac{\partial q_i}{\partial P_i} = 0$$

- if the firms collude, then the first order conditions are

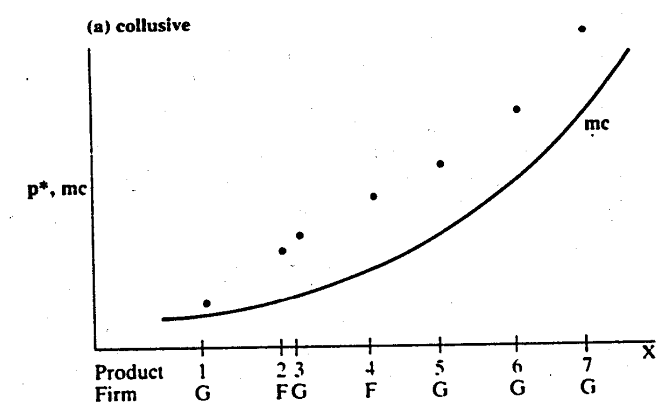
$$q_i + [P_i - mc(x_i)]\frac{\partial q_i}{\partial P_i} + [P_{i+1} - mc(x_{i+1})]\frac{\partial q_{i+1}}{\partial P_i} = 0$$

Equilibrium and Intuition

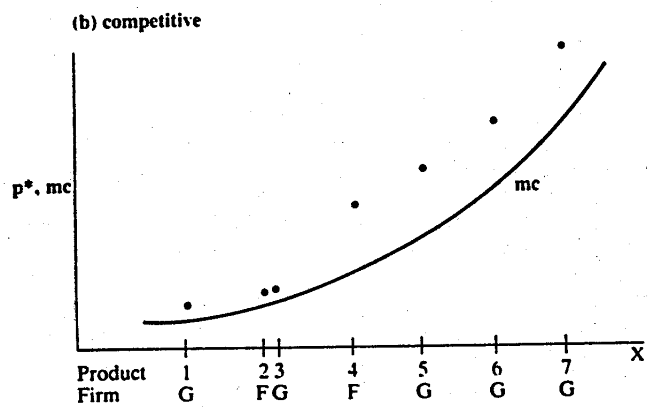
- solve the system of demand and pricing equations to obtain the reduced form

$$p = p^*(x, H, \gamma, V_{max}, \delta, \mu)$$
$$q = q^*(x, H, \gamma, V_{max}, \delta, \mu)$$

- intuition of the model



Intuition



- if products are close and firms compete, then prices are close to marginal cost
- if products are close and firms collude, then prices are above marginal cost

Econometrics

- each product is assigned a quality depending upon β and physical characteristics z
- products are ordered in terms of quality, from highest to lowest
- given product qualities and rankings, solve for the predicted values of prices and quantities
- plug the predicted values (as a function the parameters) into the likelihood function

Data

- here we need to aggregate sales of models with the same observable characteristics into a single product category
  - number of models: $80 - 85$; number of products: $140 - 150$
  - model characteristics: weight, length, horsepower, engine type and a body-type dummy
- price: list prices as of mid-April in the model year

Empirical Results

- hypothesis testing

	Collusion	Nash-Competition
1954	not rejected	rejected
1955	rejected	not rejected
1956	not rejected	rejected

- striking results: estimates across the three years are very similar
  - 1954 and 1956 results come from the Collusion specification, the 1955 estimates from the Nash-competition specification
  - the distinct features of the 1955 model year are captured by a change in behavioral assumption, not by changes in the estimated parameters

Remarks

- the conventional approach to studying market power was to look for explicit indicators of market power, for example, price-cost margins; problem with this approach is that accounting data yields poor proxies for price-cost margins

- the Bresnahan approach specifies and estimates a structural model of demand and supply, the supply equation includes a term for the demand elasticity in the presence of market power, estimating this term reveals exercise of market power and how it may vary over the sample

BLP (1995)

- goal: provide a framework for obtaining estimates of demand and cost parameters for a class of oligopolistic differentiated good markets using only aggregate data on product shares and prices

- extends the literature in two important ways
  - relax the strong functional form assumptions that restrict the substitution pattern
  - accounts for the endogeneity of prices

Data

- product characteristics: number of cylinders, number of doors, weight, engine displacement, horsepower, length, width, wheelbase, EPA miles per gallon rating, and indicator variables for whether the car has front wheel drive, automatic transmission, power steering and air conditioning

- price is list retail price for the base model, measured in 1983 dollar, quantity is sales in the US

- all models marketed during the 20 year period from 1971 to 1990, treating model/year as an observation, total sample size is 2217, number of distinct models is 997

- additional data: price of gasoline, number of HH in US, etc.

Data Overview

DESCRIPTIVE STATISTICS

Year	No. of Models	Quantity	Price	Domestic	Japan	European	HP/Wt	Size	Air	MPG	MP$
1971	92	86.892	7.868	0.866	0.057	0.077	0.490	1.496	0.000	1.662	1.850
1972	89	91.763	7.979	0.892	0.042	0.066	0.391	1.510	0.014	1.619	1.875
1973	86	92.785	7.535	0.932	0.040	0.028	0.364	1.529	0.022	1.589	1.819
1974	72	105.119	7.506	0.887	0.050	0.064	0.347	1.510	0.026	1.568	1.453
1975	93	84.775	7.821	0.853	0.083	0.064	0.337	1.479	0.054	1.584	1.503
1976	99	93.382	7.787	0.876	0.081	0.043	0.338	1.508	0.059	1.759	1.696
1977	95	97.727	7.651	0.837	0.112	0.051	0.340	1.467	0.032	1.947	1.835
1978	95	99.444	7.645	0.855	0.107	0.039	0.346	1.405	0.034	1.982	1.929
1979	102	82.742	7.599	0.803	0.158	0.038	0.348	1.343	0.047	2.061	1.657
1980	103	71.567	7.718	0.773	0.191	0.036	0.350	1.296	0.078	2.215	1.466
1981	116	62.030	8.349	0.741	0.213	0.046	0.349	1.286	0.094	2.363	1.559
1982	110	61.893	8.831	0.714	0.235	0.051	0.347	1.277	0.134	2.440	1.817
1983	115	67.878	8.821	0.734	0.215	0.051	0.351	1.276	0.126	2.601	2.087
1984	113	85.933	8.870	0.783	0.179	0.038	0.361	1.293	0.129	2.469	2.117
1985	136	78.143	8.938	0.761	0.191	0.048	0.372	1.265	0.140	2.261	2.024
1986	130	83.756	9.382	0.733	0.216	0.050	0.379	1.249	0.176	2.416	2.856
1987	143	67.667	9.965	0.702	0.245	0.052	0.395	1.246	0.229	2.327	2.789
1988	150	67.078	10.069	0.717	0.237	0.045	0.396	1.251	0.237	2.334	2.919
1989	147	62.914	10.321	0.690	0.261	0.049	0.406	1.259	0.289	2.310	2.806
1990	131	66.377	10.337	0.682	0.276	0.043	0.419	1.270	0.308	2.270	2.852
All	2217	78.804	8.604	0.790	0.161	0.049	0.372	1.357	0.116	2.099	2.086

- number of products rises from 72 in 1974 to high of 150 in 1988, sales per model trend down

Data Overview

- list prices have risen almost 50 percent during the 1980s but characteristics are also changing so not clear what is happening to cost per car with fixed characteristics
- HP/weight trended down and then up, mostly due to changes in weight, fuel efficiency trends up
- air conditioning is increasingly part of the base model
- market share of domestics fall from 93% to 68%, mostly to Japanese models

OLS and IV Logit Results

Results with Logit Demand and Marginal Cost Pricing
(2217 Observations)

Variable	OLS Logit Demand	IV Logit Demand	OLS ln(*price*) on *w*
Constant	−10.068	−9.273	1.882
	(0.253)	(0.493)	(0.119)
*HP/Weight**	−0.121	1.965	0.520
	(0.277)	(0.909)	(0.035)
Air	−0.035	1.289	0.680
	(0.073)	(0.248)	(0.019)
MP$	0.263	0.052	—
	(0.043)	(0.086)	
*MPG**	—	—	−0.471
			(0.049)
*Size**	2.341	2.355	0.125
	(0.125)	(0.247)	(0.063)
Trend	—	—	0.013
			(0.002)
Price	−0.089	−0.216	—
	(0.004)	(0.123)	
No. Inelastic Demands	1494	22	*n.a.*
(+/− 2 *s.e.'s*)	(1429–1617)	(7–101)	
R^2	0.387	*n.a.*	.656

OLS and IV Logit Results

- OLS estimates
  - most of the estimates have the right sign but not very precisely estimated
  - price coefficient is implausibly small: 1494 of the 2217 models have inelastic demands, which is not consistent with profit-maximization
  - 61 percent of the variance in mean utility is due to unobserved product characteristics
- IV results
  - all characteristics enter positively and significantly (except for MP$)
  - price coefficient increases: products with higher unobserved quality sell for higher prices
  - number of products with inelastic demands drops to 22

Random Coefficient Model with Pricing Equations

Estimated Parameters of the Demand and Pricing Equations: BLP Specification, 2217 Observations

Demand Side Parameters	Variable	Parameter Estimate	Standard Error	Parameter Estimate	Standard Error
Means ($\bar{\beta}$'s)	*Constant*	−7.061	0.941	−7.304	0.746
	HP/Weight	2.883	2.019	2.185	0.896
	Air	1.521	0.891	0.579	0.632
	MP$	−0.122	0.320	−0.049	0.164
	Size	3.460	0.610	2.604	0.285
Std. Deviations (σ_β's)	*Constant*	3.612	1.485	2.009	1.017
	HP/Weight	4.628	1.885	1.586	1.186
	Air	1.818	1.695	1.215	1.149
	MP$	1.050	0.272	0.670	0.168
	Size	2.056	0.585	1.510	0.297
Term on Price (α)	$\ln(y-p)$	43.501	6.427	23.710	4.079
Cost Side Parameters					
	Constant	0.952	0.194	0.726	0.285
	$\ln(HP/Weight)$	0.477	0.056	0.313	0.071
	Air	0.619	0.038	0.290	0.052
	$\ln(MPG)$	−0.415	0.055	0.293	0.091
	$\ln(Size)$	−0.046	0.081	1.499	0.139
	Trend	0.019	0.002	0.026	0.004
	$\ln(q)$			−0.387	0.029

- the standard deviations of the random coefficients are quite important

Substitution Patterns

A Sample from 1990 of Estimated Own- and Cross-Price Semi-Elasticities: Based on Table IV (CRTS) Estimates

	Mazda 323	Nissan Sentra	Ford Escort	Chevy Cavalier	Honda Accord	Ford Taurus	Buick Century	Nissan Maxima	Acura Legend	Lincoln Town Car	Cadillac Seville	Lexus LS400	BMW 735i
323	−125.933	1.518	8.954	9.680	2.185	0.852	0.485	0.056	0.009	0.012	0.002	0.002	0.000
Sentra	0.705	−115.319	8.024	8.435	2.473	0.909	0.516	0.093	0.015	0.019	0.003	0.003	0.000
Escort	0.713	1.375	−106.497	7.570	2.298	0.708	0.445	0.082	0.015	0.015	0.003	0.003	0.000
Cavalier	0.754	1.414	7.406	−110.972	2.291	1.083	0.646	0.087	0.015	0.023	0.004	0.003	0.000
Accord	0.120	0.293	1.590	1.621	−51.637	1.532	0.463	0.310	0.095	0.169	0.034	0.030	0.005
Taurus	0.063	0.144	0.653	1.020	2.041	−43.634	0.335	0.245	0.091	0.291	0.045	0.024	0.006
Century	0.099	0.228	1.146	1.700	1.722	0.937	−66.635	0.773	0.152	0.278	0.039	0.029	0.005
Maxima	0.013	0.046	0.236	0.256	1.293	0.768	0.866	−35.378	0.271	0.579	0.116	0.115	0.020
Legend	0.004	0.014	0.083	0.084	0.736	0.532	0.318	0.506	−21.820	0.775	0.183	0.210	0.043
TownCar	0.002	0.006	0.029	0.046	0.475	0.614	0.210	0.389	0.280	−20.175	0.226	0.168	0.048
Seville	0.001	0.005	0.026	0.035	0.425	0.420	0.131	0.351	0.296	1.011	−16.313	0.263	0.068
LS400	0.001	0.003	0.018	0.019	0.302	0.185	0.079	0.280	0.274	0.606	0.212	−11.199	0.086
735*i*	0.000	0.002	0.009	0.012	0.203	0.176	0.050	0.190	0.223	0.685	0.215	0.336	−9.376

Note: Cell entries i, j, where i indexes row and j column, give the percentage change in market share of i with a $1000 change in the price of j.

Remarks

- cross-price elasticities are large for cars with similar characteristics
- magnitudes of the impact of price increases of the higher price cars are much smaller than they are for the lower-priced cars
- patterns seem plausible: Lexus is closest substitute for BMW 735, Accord is the closest substitute for Taurus

Markups

A SAMPLE FROM 1990 OF ESTIMATED PRICE-MARGINAL COST MARKUPS AND VARIABLE PROFITS: BASED ON TABLE 6 (CRTS) ESTIMATES

	Price	Markup Over MC $(p - MC)$	Variable Profits (in $'000's) $q*(p - MC)$
Mazda 323	$5,049	$ 801	$18,407
Nissan Sentra	$5,661	$ 880	$43,554
Ford Escort	$5,663	$1,077	$311,068
Chevy Cavalier	$5,797	$1,302	$384,263
Honda Accord	$9,292	$1,992	$830,842
Ford Taurus	$9,671	$2,577	$807,212
Buick Century	$10,138	$2,420	$271,446
Nissan Maxima	$13,695	$2,881	$288,291
Acura Legend	$18,944	$4,671	$250,695
Lincoln Town Car	$21,412	$5,596	$832,082
Cadillac Seville	$24,353	$7,500	$249,195
Lexus LS400	$27,544	$9,030	$371,123
BMW 735*i*	$37,490	$10,975	$114,802

Remarks and Conclusions

- remarks on the markup
  - average markup is $3,753 and average ratio of markup to retail price is .239
  - patterns are plausible: markups are higher on higher-priced models
- conclusions
  - price endogeneity matters
  - allowing for more flexible utility specifications generates a more realistic picture of demand and equilibrium