Graduate IO: Session 2

September 20, 2015

Before Start

- please check/sign on the attendance sheet
- important: send me your email addresses (required if you are enrolled)
  - my email address: *lu.zhentong@mail.shufe.edu.cn*
  - problem set 1 will be sent out this week
  - readings for the next week will be sent before Wednesday
- group for the presentation/referee report
  - in principle 2 students
  - send me your group by Sunday
- start to look for data if you want to write an empirical IO paper

Agenda

- product differentiation: basic models
  - fixed number of products
  - turnpike model
- horizontal differentiation
  - linear city model
  - circular city model
  - Dixit-Stiglitz-Spence Model

Product Differentiation

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- most markets are differentiated product markets: products of different firms are not perfect substitutes
- our models of these markets are among the most realistic and useful of all models in IO
- two class of models: horizontal differentiation and vertical differentiation
  - we mostly focus on horizontal differentiation for now

Horizontal Differentiation

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- key attributes on which consumers differ: product characteristics or location
- examples: films, beer, cars, books, cereals, ice cream flavors, Starbucks by geographic location

Vertical Differentiation

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Vertical Differentiation

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- key attributes: quality, durability
- examples: computers, diamonds, batteries
- in general, a product is a bundle of vertical and horizontal attributes: a car is a bundle of certain amount of horsepower (vertical), color, weight, and size (horizontal)

Fixed Number of Products: Demand

- a market with two products with demand functions

$$q_1(p_1, p_2) = a - bp_1 + cp_2$$

$$q_2(p_1, p_2) = a - bp_2 + cp_1$$

  - two goods are substitutes if $c > 0$, complement if $c < 0$
  - own-price effect has to be larger than cross-price effect: $b > |c|$

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  - own-price effect has to be larger than cross-price effect: $b > |c|$

- invert the demand system

$$p_1(q_1, q_2) = \alpha - \beta q_1 - \gamma q_2$$

$$p_2(q_1, q_2) = \alpha - \beta q_2 - \gamma q_1$$

where $a = \frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}$, $b = \frac{\beta}{\beta^2-\gamma^2}$ and $c = \frac{\gamma}{\beta^2-\gamma^2}$

Fixed Number of Products: Pricing Game

- firm 1 chooses price to maximize

$$\pi_1(p_1, p_2) = (a - bp_1 + cp_2)\,p_1$$

and we can obtain

$$p_1 = \frac{a + cp_2}{2b}$$

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- solving for the equilibrium

$$p_1^* = p_1^* = \frac{a}{2b - c}$$

Fixed Number of Products: Quantity Game

- firm 1 chooses quantity to maximize

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- best replies

$$q_1 = \frac{\alpha - \gamma q_2}{2\beta}, \; q_2 = \frac{\alpha - \gamma q_1}{2\beta}$$

- equilibrium

$$q_1^* = q_2^* = \frac{\alpha}{2\beta + \gamma}$$

Pricing Game VS Quantity Game

- basic question: how do quantities and prices differ in the pricing game versus the quantity game?

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- basic question: how do quantities and prices differ in the pricing game versus the quantity game?
- answer: substituting for a, b and c, one can show that price is higher (and quantity is lower) in the capacity game than in the pricing game
  - the difference is smaller when products are more differentiated, i.e., γ goes to zero

Strategic Complements VS Substitutes

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- slope of the best reply functions is a key issue in many competitive situations
- a game is supermodular if $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} > 0$, this is the case when (i) firms choose prices and goods are substitutes, (ii) firms choose quantities and goods are complements

The Turnpike Model

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- model setup
  - one mile private road from A to B
  - N individuals own segments of the road, each owner installs a toll booth to collect a toll for her segment
  - demand: $D(P) = a - bP$ where $P = \sum_{i=1} p_i$
  - costs are zero and owners set tolls simultaneously

Equilibrium

- owner i chooses p_i to maximize

$$\pi\left(p_i, p_{-i}\right) = p_i\left(a - bp_i - b\sum_{i \neq j} p_j\right)$$

- differentiating and solving for p_i

$$p_i = \frac{a - b\sum_{i \neq j} p_j}{2b}$$

- imposing sysmmetry

$$p^* = \frac{a}{\left(N+1\right)b}$$

$$P^* = \frac{aN}{\left(N+1\right)b},\ Q^* = \frac{a}{N+1},\ \pi^* = \frac{a^2}{b\left(N+1\right)^2}$$

Implications

- what happens as N gets large?
  - P^* increases to $\frac{a}{b}$, Q^* goes to zero
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- conclusion: consumers are better off with a monopoly than competition!
  - think of $Q(P)$ as the size of the pie and $p_i Q(P)$ as individual i's share of the pie
  - at the monopoly price, the first-order effect of an increase in p_i on the size of the pie is zero
  - but an increase in p_i increases firm i's share of the pie
  - so each individual has an incentive to raise its toll, battle over higher shares destroys the pie!

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- applications: console + games, operating system + software

Hotelling Model of Product Differentiation

- this model allows us to study product location and variety
- one mile long beach: $0 =$left end point, $1 =$right end point
- $M = 1000$ people are distributed uniformly along the beach: fraction of bathers in any section of the beach of length z is $zM = 1000z$
- location of bather x is measured relative to 0
- two ice-cream vendors are located at either end of the beach
- cost of walking: t times the distance squared

Preferences

- utility function

$$u(x) = \begin{cases} s - p_1 - tx^2 & \text{if } x \text{ purchase from 1} \\ s - p_2 - t(1-x)^2 & \text{if } x \text{ purchase from 2} \\ 0 & \text{otherwise} \end{cases}$$

where s measures the utility for consuming an ice cream cone

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  where s measures the utility for consuming an ice cream cone

- total cost of buying from vendor 1 is $p_1 + tx^2$, total cost of buying from vendor 2 is $p_2 + t(1-x)^2$
- if prices are the same (and s is sufficiently large), then each bather buys from the vendor who is closer

Local Monopoly

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- no overlap in the market coverage of the two vendors when they set price equal to the monopoly price
- what is the monopoly price?
  - need to compute demand: marginal consumer $\tilde{x}$ is indifferent between buying from vendor 1 and not buying

$$s - p_1 - t\tilde{x}^2 = 0 \Rightarrow \tilde{x} = \sqrt{(s - p_1)/\, t}$$

  - vendor 1's demand at p_1 is $\tilde{x}$ and it chooses price to maximize

$$\pi(p_1) = p_1 \left(\sqrt{(s - p_1)/\, t} \right) M$$

Local Monopoly: Solution

- differentiating and solving the FOC yields the monopoly price

$$P^M = \frac{2s}{3}$$

- need to check that $\tilde{x}$ is indeed less than $\frac{1}{2}$, i.e., no overlap

$$\tilde{x}\left(P^M\right) < \frac{1}{2} \text{ iff } s < \frac{3t}{4}$$

- problem for vendor 2 is symmetric: demand is $1 - \tilde{x}\left(P^M\right)$

Duopoly

- together the two vendors cover the market so the marginal consumer is indifferent between buying from vendor 1 or vendor 2

$$p_1 + t\tilde{x}^2 = p_2 + t\left(1 - \tilde{x}\right)^2$$

$$\Rightarrow \tilde{x} = \frac{(p_2 - p_1 + t)}{2t}$$

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- demand for vendor 1 decreases in own price and increases in rival's price
- vendor 1 chooses its price to maximize

$$\pi\left(p_1\right) = p_1 \tilde{x} M = p_1 \left[\left(p_2 - p_1 + t\right)/\, 2t\right] M$$

- differentiating and solving for vendor 1's best reply yields

$$p_1 = \frac{p_2 + t}{2}$$

Equilibrium

- solving vendor 2's problem yields

$$p_2 = \frac{p_1 + t}{2}$$

Equilibrium

- solving vendor 2's problem yields

$$p_2 = \frac{p_1 + t}{2}$$

- intersection of the best replies

$$p_1 = p_2 = t$$

- marginal consumer is $\tilde{x} = 1/2$
- equilibrium profits are $\pi_1 = \pi_2 = t$

Extension: Location Choice

- we can endogenize the location of the vendors using a two-stage game: vendors first choose their location and then compete in prices
- question: will vendors locate as far from each other as possible (maximal differentiation) or as close as possible (minimal differentiation)?

Extension: Location Choice

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- question: will vendors locate as far from each other as possible (maximal differentiation) or as close as possible (minimal differentiation)?

- assume vendor 1 is located at point a and vendor 2 is located at $1 - b$ where $1 - b \geq a$
  - $a + b = 1$ is minimal differentiation
  - $a + b = 0$ is maximal differentiation

Location Choice

- stage 2: given any pair of locations $(a, 1-b)$, demand functions are

$$D_1(p_1, p_2) = \tilde{x} = a + \frac{1-a-b}{2} + \frac{p_2 - p_1}{2t(1-a-b)}$$

$$D_2(p_1, p_2) = 1 - \tilde{x} = b + \frac{1-a-b}{2} + \frac{p_1 - p_2}{2t(1-a-b)}$$

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- Nash equilibrium prices given locations are

$$p_1^*(a, b) = t(1-a-b)\left(1 + \frac{a-b}{3}\right)$$

$$p_2^*(a, b) = t(1-a-b)\left(1 + \frac{b-a}{3}\right)$$

Location Choice (Cont.)

- substituting the equilibrium prices into the vendors' profit functions yields the reduced form profit functions for $i = 1, 2$

$$\pi_i(a, b) = p_i^*(a, b)\, D_i\left(p_1^*(a, b), p_2^*(a, b)\right)$$

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- given b, firm 1 chooses a to maximize profits

$$\begin{aligned} \frac{\partial \pi_1(a, b)}{\partial a} &= \frac{\partial p_1^*}{\partial a} D_1 + p_1^* \left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_1} \frac{\partial p_1^*}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} \right] \\ &= p_1^* \left[\frac{\partial D_1}{\partial a} + \frac{\partial D_1}{\partial p_2} \frac{\partial p_2}{\partial a} \right] \end{aligned}$$

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- verify that $\frac{\partial D_1}{\partial a} = \frac{3-5a-b}{6(1-a-b)}$ and $\frac{\partial D_1}{\partial p_2}\frac{\partial p_2}{\partial a} = \frac{a-2}{3(1-a-b)}$

Location Choice (Cont.)

- combining the expressions, we obtain

$$\frac{\partial \pi_1(a,b)}{\partial a} < 0$$

- implication: vendor 1 wants to locate as far from vendor 2 as possible, the same is true for vendor 2
  - subgame perfect equilibrium is $a^* = b^* = 0$ and $p_1^* = p_2^* = t$

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- implication: vendor 1 wants to locate as far from vendor 2 as possible, the same is true for vendor 2
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- basic conflict: locate where demand is (move to the center) $\leftrightarrow$ stay away from competition (go to the ends)
  - depends on travel costs: quadratic travel costs $\rightarrow$ the second force dominates

Salop's Circular City

- consumers are distributed uniformly on a circle with circumference 1
- each consumer demands one unit, willingness to pay is s
- transport costs are linear: t per unit of distance traveled
- N firms are symmetrically located around the circle at distance $\frac{1}{N}$ apart
- entry cost is f, production cost is c per unit

Demand

- assumption: t is sufficiently large that a symmetric pure strategy equilibrium exists in which firms service the neighborhoods around their locations
  - each firm i competes only with firms $i-1$ and $i+1$
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$$p_i + tx = p + t\left(\frac{1}{n} - x\right)$$

- thus demand for firm i is

$$D\left(p_i, p\right) = 2x = t^{-1}\left(p + \frac{t}{n} - p_i\right)$$

Equilibrium

- firm i chooses p_i to maximize

$$\pi_i\left(p_i, p\right) = \left(p_i - c\right) t^{-1} \left(p + \frac{t}{n} - p_i\right)$$

Equilibrium

- firm i chooses p_i to maximize

$$\pi_i(p_i, p) = (p_i - c)\, t^{-1} \left(p + \frac{t}{n} - p_i\right)$$

- best reply is

$$p_i = \frac{1}{2}\left(p + c + \frac{t}{n}\right)$$

Equilibrium

- firm i chooses p_i to maximize

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- best reply is

$$p_i = \frac{1}{2}\left(p + c + \frac{t}{n}\right)$$

- imposing symmetry yields the symmetric equilibrium price and profit

$$p^* = \frac{t}{n} + c,\ \pi^* = \frac{t}{n^2}$$

- equilibrium number of firms: zero profit condition

$$\pi^* - f = 0 \Rightarrow n^* = \sqrt{\frac{t}{f}}$$

Does the market generate too much or too little variety?

- social planner's problem: choose n to minimize sum of fixed costs and consumer's transport cost

$$\min_n \left\{ nf + t\,(2n) \int_0^{\frac{1}{2n}} x dx \right\} = \min_n \left\{ nf + \frac{t}{4n} \right\}$$

  - remark: as long as market is covered, total surplus is fixed at $(s - c)$

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- solution

$$\tilde{n} = \sqrt{\frac{t}{4f}} = \frac{n^*}{2}$$

- conclusion: market generates too much product variety
  - intuition: entrants do not take into account the impact of their entry on profits of incumbents
  - equi-distant spacing of products can be justified if transport costs are quadratic

Dixit-Stiglitz-Spence Model

- consumers have CES preferences over a numeraire good q_0 and n differentiated goods

$$U\left(q_0, \left(\sum_{i=1}^{n} q_i^{\sigma}\right)^{1/\sigma}\right),\ \sigma \leq 1$$

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- each firm produces one good, production involves a fixed cost f and marginal cost c

Demand

- substituting for q_0 using the budget constraint, demands for the differentiated products are given by the solution to the system of n equations

$$\frac{\partial U}{\partial q_0} p_i - \frac{\partial U}{\partial q_i} \left(\sum_{i=1}^{n} q_i^{\sigma} \right)^{\frac{1}{\sigma}-1} q_i^{\sigma-1} = 0, \; i = 1, ..., n$$

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- each good competes with every other good, suppose n is sufficiently large

$$q_i \approx k p_i^{-\frac{1}{1-\sigma}}$$

where $k \equiv \frac{\partial U}{\partial q_0} \Big/ \frac{\partial U}{\partial q_i} \left(\sum_{i=1}^{n} q_i^{\sigma}\right)^{\frac{1}{\sigma}-1}$ is approximately a constant

Supply

- firm i chooses p_i to solve

$$\pi_i\left(p_i\right) = \left(p_i - c\right) q_i\left(p_i\right) - f$$

- differentiating and solving

$$\left(p_i - c\right)\left(-\frac{1}{1-\sigma}\right) k p_i^{-\frac{1}{1-\sigma}-1} + k p_i^{-\frac{1}{1-\sigma}} = 0$$

$$\Rightarrow p_i = \frac{c}{\sigma}$$

  - equilibrium prices are independent of n

Equilibrium

- to determine n^*, q^*, we can use zero-profit condition and FOC

$$\left(\frac{c}{\sigma} - c\right) q^* = f$$

$$\frac{\partial U}{\partial q_0} p_i = \frac{\partial U}{\partial q_i} n^{\frac{1}{\sigma} - 1}$$

- solution to these two equations turns out to be the solution to the social planner's problem